# PHYSICS

## A Magnetostriction Oscillator Producing Intense Audible Sound and Some Effects Obtained*

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Apparatus is developed capable of pushing the magnetostrictive oscillation of nickel to the limit set by its mechanical strength. The intense audible sound produced is introduced under water from beneath the surface, continuous operation of the vibrator being secured by spray cooling. The interiors of corks are charred to cinder by the sound. A fountain 6 cm high forms on the surface of water, and is proved to be due to pumping action of the reciprocating rod and *not* to radiation pressure. Erosion of metals and of glass by cavitation and resultant water hammer is obtained. Colloids of oil and of carbon in water are produced. Curious fatigue effects in glass are observed. Whitened appearance and erratic motions of air bubbles released in water in the presence of the sound are observed, and theory is developed to explain these and the "atomization" of liquids. Flattening of a drop of liquid against a vibrator is observed and attributed to sonic attraction and to the reaction of sonic radiation. Frogs, fish, larvae, and water fleas are killed by the sound. Bacteria are killed so quickly that partial sterilization of milk in continuous process is effected. Striations in a Kundt tube that are parallel to its axis are produced, and an explanatory theory developed.

### INTRODUCTION

THE use of a rod or tube of magnetic material to stabilize an oscillating electric circuit was first proposed by G. W. Pierce,[1] and such rods were employed by him in the production of standards of frequency in the upper audible range. Other investigators[2] have used such rods for various purposes, but so far as the writer knows there have been no investigations at great amplitudes of vibration.

Quartz crystals, actuated by intense electrostatic fields through the piezoelectric effect, have been employed by Langevin, Boyle, Wood and Loomis, and others to produce within liquids very strong, ultrasonic vibrations, some effects of which they have noted.[3] Very intense audible sound has never been produced by use of resonant crystals in longitudinal vibration, because crys-

* Containing parts of dissertation, Newton Gaines, University of Texas, 1931.

[1] Pierce, Proc. Am. Acad. Arts and Sci. **63,** 1 (1928); reprinted, Proc. Inst. Radio Eng. **17,** 42 (1929).

[2] Black, Proc. Am. Acad. Arts and Sci. **63,** 49 (1928); Lange and Myers, Proc. Inst. Radio Eng. **17,** 1687 (1929).

[3] Boyle, Sci. Prog. **23,** 75 (1928); Wood and Loomis, Phil. Mag. **4,** 417 (1927); Harvey and Loomis, Nature **121,** 622 (1928) and J. of Bacter. **17,** 373 (1929); Schmidt, Johnson, and Olson, J. Am. Chem. Soc. **51,** 370 (1929); Schmidt, Protoplasma **7,** 332 (1929); Richards, J. Am. Chem. Soc. **51,** 1724 (1929); Abello, Phys. Rev. **31,** 1083 (1928); Chambers and Harvey, J. Morph. and Physiol. **52,** 155 (1931).

tals thick enough to give audible sound at resonance would require extremely high voltages to make them oscillate strongly.

In the present investigation the object has been to increase the power input of resonant magnetostrictive nickel tubes to the limit as set by the mechanical properties of nickel, and then to discover and investigate properties of the intense audible sound produced by the longitudinal vibration. The research has followed somewhat that done by experimenters in the supersonic range, but has produced some results and effects so far never reported.

## The Apparatus

The apparatus for producing the intense sound and introducing it under water as developed by the author in the physical laboratory of the University of Texas has already been very briefly described.[4] A simpler and somewhat more effective form of the apparatus has been developed during the past year in the physical laboratory of Texas Christian University under grant from the American Association for the Advancement of Science. This later form of the apparatus employs but one UV-204A, 250-watt radiotron; and all power is taken in at 115 volts, 60 cycles, the direct current for the polarizing field magnet and the direct current at 2500 volts for the plate being supplied from rectifier-filters contained within the cabinet, see Figs. 1 and 2.

The vibrator consists of a cold-drawn unannealed nickel tube 3/4-inch (1.9 cm) in diameter and about 26.4 cm long, closed at the upper end by a thick nickel plate welded to it. In order to introduce sound under water from beneath, the upper half of the tube is surrounded by water as shown, the lower half only projecting into the activating coils, $L_1$ and $L_2$, a flexible water-tight connection being made between the glass vessel and the middle of the nickel tube. When thus partly submerged in water, the tube has a fundamental frequency of about 8900 cycles per second. The lower half of the tube contains a longitudinal slit to diminish electrical eddy currents, the heating effect of which and of hysteresis losses is still so great as to prevent the tube from operating effectively, even with its upper half under water, for longer than 1/2 minute at a time. Beyond this period, with temperature somewhere in the neighborhood of melting solder, it is observed that nickel tubes cease to give maximum response in magnetostriction oscillation.

The introduction of cooling water, however, enables the tube to be operated continuously for hours at a time. Cooling is accomplished in one of two ways, by introducing a spray of water inside the nickel tube from a glass or hard rubber nozzle introduced through the open lower end of the tube, or by piercing two small holes immediately beneath the end-plate of the vibrator thus allowing water to trickle down through the tube.

It is found that the coils when located as shown on one half of the tube prove very effective in energizing the nickel vibrator. For the purpose of producing very intense sound for the effects to be described, it is found better to connect grid coil $L_2$ so that the direction of its winding with respect to that of $L_1$ shall be as in ordinary electrical oscillator circuits and not with reverse

[4] Williams and Gaines, Jour. Infec. Diseases **47**, 485 (1930).

connections as when a magnetostrictive oscillator is to be used as a frequency stabilizing device.[1]

A fairly strong polarizing field, obtained by means of the d.c. magnet shown in the diagram, is necessary before maximum intensity of vibration can be secured. In making connections to the plate coil, care is taken that the d.c. component of its current shall aid the polarizing field.

Refer to Fig. 1: $Am_1$ is a 2.5-ampere high-frequency ammeter, protected from injury in case of breakdown of the 0.004-to-0.015 mf tuning condenser, $C_1$, by the 3000-volt, 1 mf condenser, $C_2$. $C_3$ is a grid condenser of

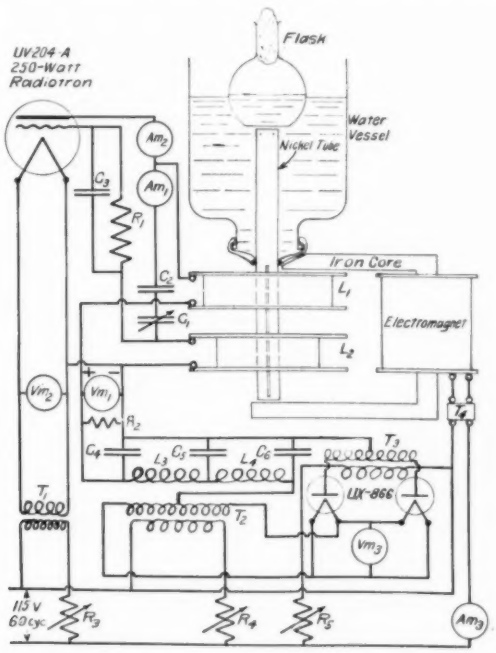


Fig. 1. Diagram of connections.

0.006 mf capacity, and $R_1$ the grid leak resistance of about 8000 ohms. Solenoids, $L_1$ and $L_2$, are wound on Bakelite spools, with openings large enough to give the nickel tube ample room. The solenoids are wound with No. 22 B. and S., enameled, cotton-covered, copper wire, being afterwards soaked in insulating compound and then baked. The d.c. resistances of $L_1$ and $L_2$ are, respectively, about 7 ohms and 3 ohms.

The rectifier-filter supplying the plate of the UV-204A radiotron consists of the step-up transformer, $T_3$, the capacitances $C_4$, $C_5$, and $C_6$ of several microfarads each, inductances $L_3$ and $L_4$ of 25 henrys each, two UX-866 rectifier tubes, filaments of which are supplied with current by transformer $T_2$. $R_2$ is a high-resistance leak. $Vm_1$ is a 2500-volt d.c. voltmeter, and $Vm_3$ an a.c. voltmeter. Transformer $T_1$ furnishes 11 volts to the filament of the radiotron, the voltage being indicated by $Vm_2$. Variable resistances $R_3$, $R_4$, and $R_5$ give control of the various currents and voltages supplied. Milliammeter

$Am_2$ indicates the d.c. plate current of the radiotron. $T_4$ is a transformer-rectifier outfit of the "B-eliminator" type. Fine tuning may be accomplished with the continuously variable part of $C_1$, but other methods of securing this final tuning are by varying the plate voltage by means of rheostat $R_5$ and by changing the strength of the polarizing field by means of a rheostat inserted on the a c. side of $T_4$.

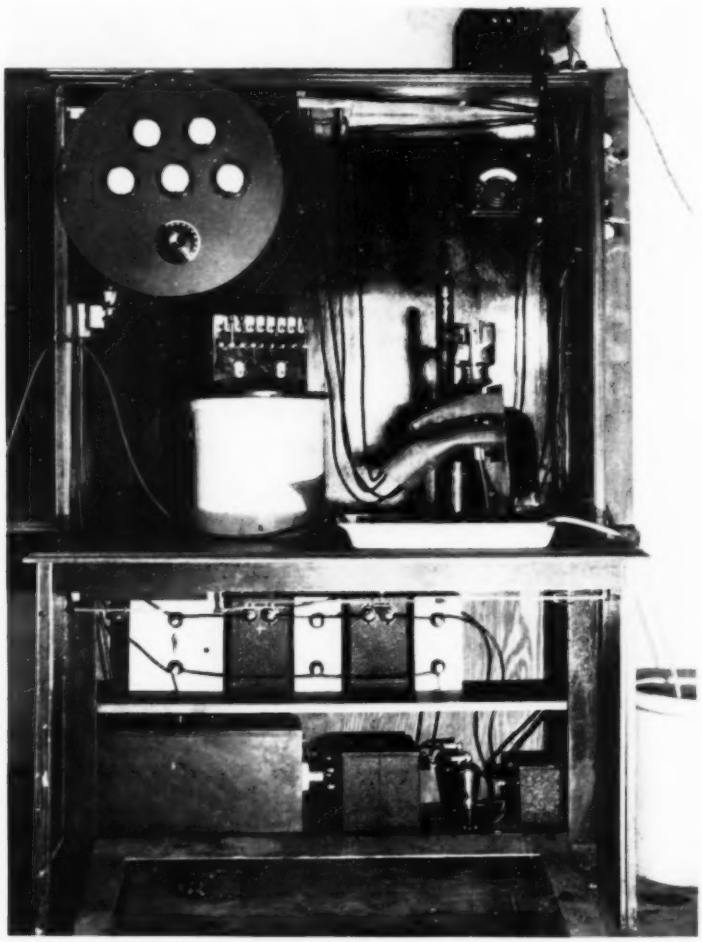

Fig. 2. Complete apparatus, door of filter cabinet open.

## Nickel Tubes Quickly Ruptured by Magnetostrictive Vibration

The apparatus above described proves sufficient to produce maximum practicable amplitude of vibration in nickel tubes, all of which are eventually broken near the middle, due to fatigue of the metal.

It should be mentioned here that the sound produced by a nickel tube vibrating undamped by water is of sufficient intensity to cause pain ulti-

mately to unprotected ears. The use, however, of small plugs of cotton in the ears has been found sufficient protection to enable the investigator to work in comfort while exposed to the sound.

Measurements of the amplitude of vibration of half of the 26.4 cm nickel tube were made with ocular micrometer, resulting in 0.01 mm when the upper half was in water, and 0.03 mm when the whole tube was in air. The approximate value of the maximum instantaneous tension, $T_m$ at the middle may be calculated. A rod or tube vibrating longitudinally in its fundamental mode has a sinusoidal distribution of tension; consequently $T_m = \pi T_a/2$, where $T_a$ is the average tension. Now $T_a = eY/L$, where $e$ is the elongation of the half of the tube, i.e., the amplitude of vibration, $L$ is the half-length,

Fig. 3. Rupture of nickel tube by magnetostrictive vibration, wall of tube 0.81 mm thick.

and $Y$ is Young's modulus. Young's modulus for nickel[5] is $32 \times 10^6$ lb./inch$^2$, and the length of the tube between the node of motion and the end is 13.2 cm; hence, for a tube vibrating in air,

$$T_m = \frac{\pi \times 0.003 \times 32 \times 10^6}{2 \times 13.2} = 11{,}500 \text{ lb./inch}^2.$$

The condition of tension at the middle of the rod alternates, of course, with a condition of compression whose maximum value must be about equal to the maximum value of tension, the stresses being approximately half the elastic limits of nickel. According to the conclusions of Woehler and Bauschinger,[6] working with wrought iron and steel respectively, fatigue rupture of these two metals may be expected after about 40 million cycles of tension and compression a little greater than half the elastic limits. To produce 40 million such cycles with the present magnetostriction apparatus at 8900

[5] Handbook of Chem. and Phys., 15th edition, p. 821.

[6] Merriman, *Mechanics of Materials*, J. Wiley and Sons, 1910, pp. 353, 381.

cycles per second requires but 75 minutes. As a matter of observation, the time actually required to break an 8900-cycle tube vibrating in air ranges from 15 to 90 minutes. The author suggests that this device for quickly fatiguing nickel might prove of value to investigators interested in the mechanical properties of this important metal and of other magnetostrictive elements and alloys. X-ray analysis of crystal structure in the vicinity of the fracture should prove interesting.

A typical fatigue crack, near the middle of a nickel vibrator, is shown in Fig. 3. This particular crack developed after 13 hours of vibration with half of the tube under water. The time of beginning of such a crack is always indicated by a decided decrease in the maximum intensity of vibration obtainable from the tube.

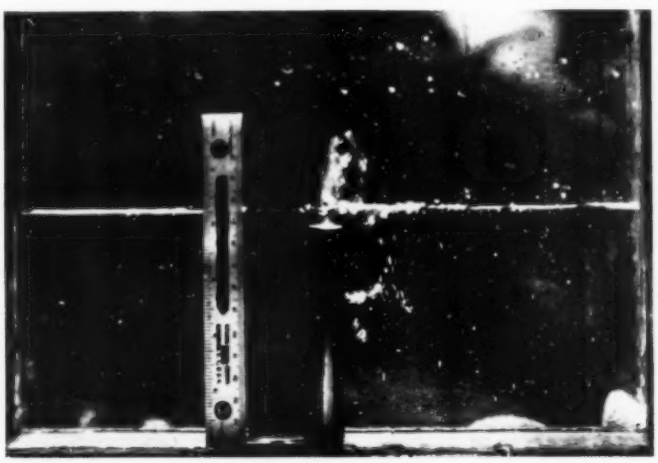

Fig. 4. Fountain of water produced above end-plate of vibrator.

## Mound or Fountain of Water Produced Above the End-Plate of Vibrator

A mound or fountain somewhat similar to that described by Wood and Loomis[3] and other workers in the ultrasonic field[7] is raised above the surface of the water directly above the closed end of the vibrating nickel tube. Fig. 4 shows the fountain, 4 or 5 cm high, produced with water 1 cm deep above the end-plate of the tube,[8] and Fig. 5 shows a mound produced above the vibrator when there is 6.5 cm of water above the end-plate. The pictures suggest that in the case of a vibrating nickel rod the mound raised is due to water moving in a vertical column, and not to radiation pressure. That such a ver-

[7] Hopwood, Jour. Sci. Instr. **6**, 34 (1929).

[8] A fountain fluctuating from 6 cm to 9 cm high can be produced under similar conditions. Tuning is rather critical for maximum height of fountain; thus, even though the fundamental frequency of the oscillating current is less than 9 kilocycles, it is found that bringing one's hand near the coils $L_1$ and $L_2$ will decidedly change the height of a fountain, an effect which suggests that the harmonics of higher order present are strong. Some of the curious effects of the vibration noted in this paper may be due to these ultrasonic vibrations.

tical current does exist is shown by the motion of ink released within the water anywhere in the vicinity of the end-plate of the vibrating tube.

The simple test next described seems to the writer to indicate decisively that no appreciable mound due to radiation pressure is present on the surface of the water. With liquid about 8 cm deep above the end-plate of the vibrating tube, the water in the trough is given a motion of about 15 cm per second

Fig. 5. Mound produced above end-plate when latter is 6.5 cm under water.

from left to right past the end of the vertical tube. The mound is observed to shift to the right, as shown in Fig. 6. Similarly, when the motion imparted to the water is from right to left, the bump is shifted to the left. There is no indication of a mound directly above the end of the vibrator, and the fine mist of tiny "bubbles,"[9] which indicates the location of the stream of water

[9] That the "bubbles" comprising this fine mist, which is plainly shown in Figs. 5 and 6, are really cavitations and contain very little air, is indicated by instantaneous disappearance of all trace of the mist when the oscillator is suddenly stopped and by the fact that only 11 $cm^3$ of air is caught when a water-filled test tube is inverted and held for one hour in the water over the center of the reciprocating end-plate.

moving upward from the end-plate, points unmistakably to the mound. When the lateral motion of the water is increased to several feet per second, the mound fails to appear on the surface of the water and the remnant of the stream of tiny bubbles is seen to bend downwards, indicating that the column of moving water is joining in the general circulation within the trough.

The author reasons that sound, because of its high velocity in water, would not be appreciably changed from its vertical course by the comparatively slow, horizontal motion of the water, therefore that the mound, if it had been due to radiation pressure, would have remained almost directly above the end-plate of the tube.

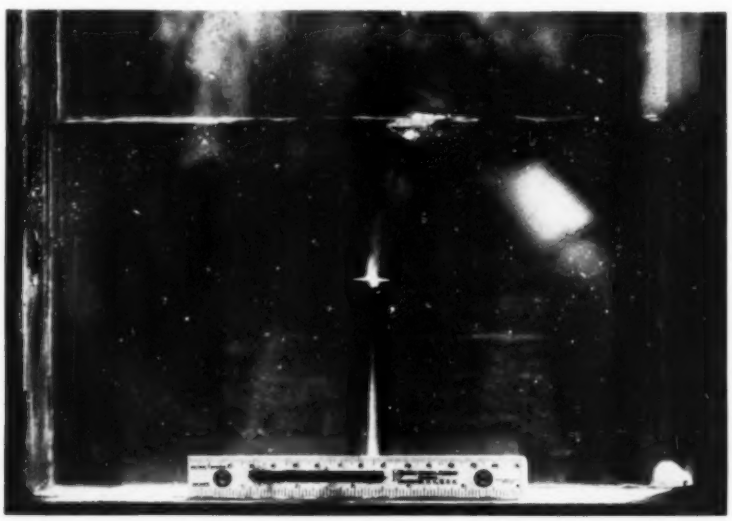

Fig. 6. Shift of mound due to motion of water toward right.

In view of the above, the author is convinced that the mound or fountain of water produced by his apparatus is not due appreciably to radiation pressure but to a column of water moving vertically upward from the end-plate of the vibrating tube. It should be of interest, he thinks, for some experimenter possessing a crystal oscillator to try the experiment of giving sideways motion to the oil above the crystal. In event the mound of oil is due partly to a vertical convection current as well as to radiation pressure, two mounds should appear when the oil is moving sideways.

## How the Reciprocating End-Plate Acts as a Pump

The existence of a column of water moving upward from the reciprocating end-plate of the nickel tube requires that something equivalent to valve action be present, since such action is essential to the operation of a reciprocating pump.

Valve action in the present case doubtless results from the presence of cavitation at the end of the tube. When the mound begins to appear on the

surface of the water, cavitation is invariably observed on the end-plate and a clattering noise suggesting water-hammer is always heard. The cavitation is in the form of a white, hazy, fairly symmetrical star consisting of eight or more radial lines converging into a white central "bubble"[10] at the center of the end-plate. A pattern is etched in the metal beneath this cavitation, as will be described in the following section, see Fig. 9.

Cavitation is to be expected because the acceleration of the vibrating end-plate is very great. Its maximum value is given by the formula, $Y=4\pi^2n^2A$, where $n$ is the frequency and $A$ the amplitude. At 8900 cycles per second and the measured amplitude under water of 0.01 mm, the maximum acceleration becomes $Y=39.48\times8900^2\times0.001=31$ kilometers per second per second. This is about one-tenth of the acceleration of the bullet of a rifle while it is moving under the influence of the powder gas in the gun barrel.

The water fails to follow the reciprocating end-plate, the force of inertia being greater than that of adhesion between water and nickel, and cavitation results. The discontinuity is presumably a vacuum containing some water vapor and a little air, and is, probably, a transient of very short duration. The presence of some air, presumably derived from that dissolved in the water, is indicated by the character of the fine mist of "bubbles" which rises with the current from the end-plate, see previous section of this article.

The fact that the cavitation does not appear over the whole of the plate, but is in the form of a star-shaped pattern, might be explained by assuming that the plate does not move as a whole but divides up into nodes and loops of motion like a Chladni plate; some observations to be given later, however, seem to oppose this assumption.

At each vibration, when the plate recedes from the water, the water at the circumference has time to rush in a short way toward the center of the plate partially to fill the cavitation. When the plate returns, the cavity probably disappears, only to reappear again the next half cycle. The thin ring of water that started toward the center of the plate continues to shrink of its own inertia until it is near the central "bubble," when it spurts upward giving rise to the vertically moving column of water. The motion of the shrinking ring of water would somewhat resemble part of the progress of the splash of a drop of mercury on a smooth glass plate as illustrated on page 23 of Professor A. M. Worthington's booklet, *The Splash of a Drop.*[11]

The maximum static pressure obtainable by this reciprocating pump was measured by holding a vertical glass tube of 4 mm bore with its lower end within a millimeter of the center of the end-plate of the vibrating tube. The

[10] A cavity in the water rather than a true gas bubble because, when vibration is suddenly stopped, it is observed that this central "bubble" together with its surrounding pattern instantly disappears, leaving no gas bubbles. Also, if the experimenter introduces a stream of small air bubbles into the "bubble" or cavitation pattern, the clattering noise disappears, which indicates that the water hammer is now being cushioned by air.

[11] Worthington, *The Splash of a Drop*, Society for Promoting Christian Knowledge, London, 1895.

water rose to a height of about 155 cm. The glass tube had at its lower end no flare or funnel such was used by W. T. Richards in his intensity gauge for ultrasonic radiation.[12]

## Erosion of Metal and of Glass by Water

A gray, star-shaped pattern quickly develops upon the surface of the nickel end-plate of a tube when vibrating under distilled water. Figs. 7 and 8 show the end-plate of the same tube after three and thirty minutes' vibration, respectively. An end-plate that has vibrated ten hours under water is shown in Fig. 9, and the pattern, which has been eroded directly beneath the radial lines of the cavitation figure, is plainly visible; the depth of the lines of this erosion pattern is as great as 0.3 mm and the depth of the central crater, which forms under the white central "bubble" of the cavitation pattern, is 0.5 mm.

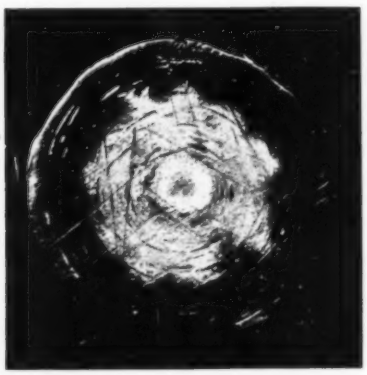

Fig. 7. Nickel end-plate after vibrating three minutes under water.

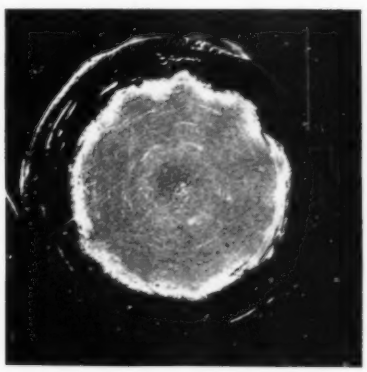

Fig. 8. Same end-plate after vibrating thirty minutes under water.

The phenomenon is evidently similar to the harmful roughening of the surface of the propeller blades of fast ships. A paper[13] published in 1919 by Sir Charles A. Parsons and Stanley S. Cook gave results of a mathematical and experimental investigation of the "possible causes of corrosion or erosion of ship propellers," with the conclusion that the phenomenon was not chemical but purely a mechanical effect being caused by cavitation in the wake of the screw propeller followed by water-hammer due to the collapse of the cavity. There being no cushion of air to break the force of the blow, the latter was very intense and sharp. Momentary pressures as high as 140 tons per square inch were measured by Parsons and Cook, the measurement involved the piercing of a thin metal plate at the small end of a hollow metal cone by the water-hammer produced when the cone was filled with water in a water-filled tank and thrust quickly downward until suddenly arrested by a block of rubber. This caused a cavity to form momentarily in the apex of the cone

[12] Richards, Proc. Nat. Acad. Sci. **15,** 310 (1929).

[13] Parsons and Cook, Engineer **127,** 427 (1919).

which cavity closed with a metallic click and sufficient force to puncture the metal plate.

In the present work, the author has made tests of the eroded and non-eroded areas of the nickel end-plates, using potentiometer and calomel cell,[14] and finds that there is no higher degree of oxidation on the eroded than on the polished surface adjacent to it, which indicates that oxidation is not the cause of the roughening.

He has also been able to recover some finely divided material which comes from the eroded area. It is found to be bright metallic nickel. Borax bead test confirms nickel. Some of this finely divided metal, magnified 50 diameters,[15] is shown in Fig. 10. Microscopic inspection of the eroded surface reveals a jagged character closely resembling that revealed in photographs by Parsons and Cook[13] of the roughened surface of a propeller.

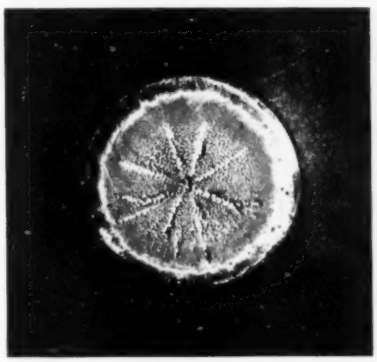

Fig. 9. A nickel end-plate after ten hours' vibration under water.

Fig. 10. Finely divided nickel (magnification 50 diameters) eroded from end-plate of vibrator.

It is evident that the above tests and observations confirm the conclusions of Parsons and Cook that such erosion is purely mechanical.

A study has been made to determine whether the shape of the erosion patterns is due to vibration of the end-plate in the manner of a Chladni plate. It seems reasonable that if the star-like patterns owe their shape to the existence of a Chladni type of vibration of the end-plate, then the thicker the plate the less pronounced should be this pattern. Tests with tubes having end-plates from 2 mm to 10 mm thick reveal no such tendency, and hence indicate that the peculiar star-like shape of the cavitation and erosion pattern cannot be explained as due to a Chladni disk type of vibration.

It is found that when disks of steel or other metal are soldered to the end-plate of a nickel tube and made to vibrate under water, erosion patterns are quickly produced, the rate of development depending upon the properties of

[14] Schoch, Am. Chem. J. **41**, 203 (1909).

[15] This microphotograph was very kindly made by Dr. F. L. Whitney, Dept. of Geology, University of Texas.

the metal. I wish to suggest that the apparatus described should prove of practical value to those interested in routine testing and research in the field of erosion especially in comparative studies of different metals proposed for ship propellers.

It is also observed that if a piece of glass is held close enough to the end-plate of a vibrating nickel tube that it touches the central "bubble" of the cavitation pattern, but not touching the end-plate itself, the surface of the glass becomes permanently whitened. Microscopic examination leads to the conclusion that the effect is due to many sharp blows such as would accompany collapse of the cavitation in contact with the glass.

## Mechanical Production of Colloids

Very fine dispersions of carbon were obtained in water, acetone, carbon tetrachloride, methyl alcohol, and other liquids by first depositing pure carbon from an acetylene flame upon the upper half of the tube, then vibrating the nickel tube under the liquid, the latter being enclosed in an inverted test tube covering the upper half of the vibrator, flexible waterproof connection being made between the mouth of the test tube and the node of motion of the vibrator. Each sol, upon test, strongly exhibited the Faraday-Tyndall effect. Evidently the soot had been sheared between the vibrating rod and the liquid into particles of colloidal size. The dispersions of carbon in water and in acetone proved to be fairly permanent, showing little change after being allowed to remain quiet for one month.

It is of interest to note that W. T. Richards has reported[3] the production of an extremely thin dispersion of glass in water, using intense sound of supersonic frequency.

The production of emulsions of two non-miscible liquids such as oil and water and mercury and water has been reported by Wood and Loomis[3] working with ultrasonic sound. These effects have been repeated by the present author using intense audible sound obtained by magnetostriction oscillation. When a beaker containing oil floating on water was dipped into the fountain above the vibrating tube until within 2 mm of the end-plate, the water became very cloudy with colloidal oil, the cloudiness remaining for several days. When a flat-bottomed test tube containing mercury and water or mineral oil was brought into contact with the vibrating end-plate, a dark gray colloid of mercury was produced which lasted for weeks.

When clean radish or turnip seed was immersed in water in a flask and the latter dipped into the fountain above the end of the vibrating nickel tube, a cloudiness was produced in the water above the seed. That the suspended substance was probably partly protein was indicated by the disagreeable odor developed when the liquid was exposed to the air for a couple of days.

A piece of kaolin was pressed directly against the submerged end-plate of the vibrating tube, the white clay rapidly disintegrated forming a dense colloid with the water. Ten days later there was still a considerable amount of kaolin in suspension.

The formation of colloids by vibration is now being carried on in collabora-

tion with Dr. Leslie A. Chambers using a device developed by us that brings the mixture to be colloidized into direct contact with the vibrating tube and end-plate in continuous flow. Dense unprotected emulsions of nujol and of olive oil in water have thus been produced that have lasted several months. This work is more fully reported in another journal.[16]

## Fatigue of Glass

An effect of strong supersonic sound reported by Wood and Loomis[3] is its tendency to cause glass exposed to it to crack, and is illustrated in their article by photograph. In this photograph, I wish to call attention to a curious tendency of the glass that has cracked away to leave rounded or oval openings.

A similar effect is obtained with the intense audible sound produced by magnetostrictive vibration. Fig. 11 shows a flask used for about five hours

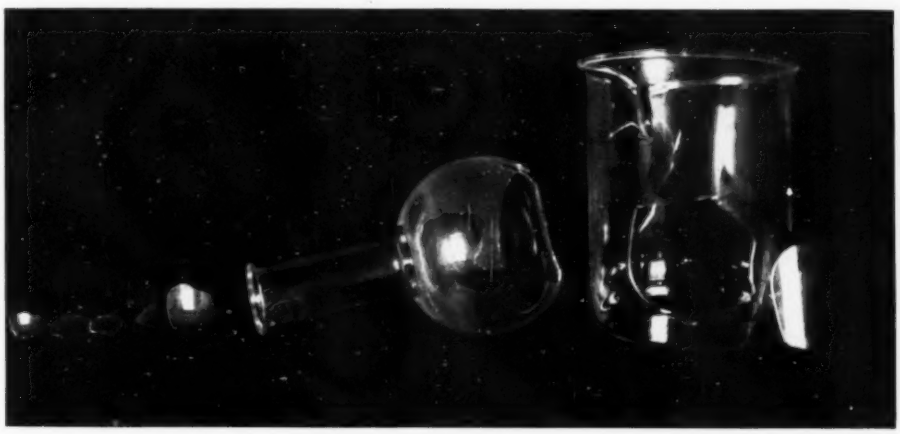

Fig. 11. Fatigue of glass. Note rounded or oval openings.

in exposing bacteria to sound, as described later. At the right is a Pyrex beaker that has been subjected to the sound. Fragments of the glass in each case were preserved, their round or oval character is apparent.

## Biological Research at Sonic Frequencies Not Complicated by Temperature Effects

The temperature rise within water exposed to the intense audible sound is so small that it may be easily controlled. When water within a flask is held above the end-plate of the vibrating tube, Fig. 1, and water containing ice is used in the vessel surrounding the flask, the water in the flask never becomes more than $5\frac{1}{2}$°C warmer than the fluid outside, even after an hour of continuous exposure to the audible sound. This ready control of temperature when audible sound is used, is of importance to research in the biological and chemical effects of sound.

[16] Chambers and Gaines, J. of Cellular and Compar. Physiology **1**, 451 (1932).

Wood and Loomis[3] have pointed out that the kinematic coefficient of viscosity of liquids increases as the square of the frequency. At frequencies of from 300,000 to 400,000 cycles per second, they observed pronounced heating effects in liquids, water in a test tube surrounded by ice water suffering a rise as great as 1°C every three seconds.

### Frogs, Fish, Larvae and Bacteria Killed by Audible Sound

The larvae (nymphs) of the Mayfly, when put into water in a test tube and held about 2 mm above the end-plate of the oscillating nickel tube, were killed within a few seconds.

Bacteria (Escherichia coli) in buffer solution in a small flask were irradiated directly above the end-plate of the nickel tube, which was vibrating under water as shown in Fig. 1. Samples were taken at the beginning and every 10 minutes for an hour. This particular piece of work was done jointly with Dr. O. B. Williams and Mr. Nathan Carpenter at the University of Texas. The results have been published in full;[4] the law of survival for the culture used was found to be $N = N_0\epsilon^{-bt}$, where $N$ is the number of survivors per cubic centimeter at any time $t$, $N_0$ is the number at the start, $\epsilon$ is the base of natural logarithms, and $b$ is a constant.

The effects of the intense audible sound upon various forms of life are now being intensively investigated at Texas Christian University in collaboration with Dr. Leslie A. Chambers, under grant for research from the American Association for the Advancement of Science, and a detailed report of the results thus far obtained is given in another journal.[16] We find that frogs, tadpoles, and fishes are quickly killed by the sound, as previously reported by workers in the ultrasonic field.[3] Formation of bubbles within the muscles, haemolysis of blood corpuscles, and other effects of this intense audible sound, are the probable causes of death.

Chambers and I find that the bacterial count of milk (Breed's standard plate method) may be reduced more than 99 percent in 45 minutes when exposed in a flask as shown in Fig. 1. Reasoning that there is much waste of sonic energy by this method of exposure, we have developed a continuous process for the treatment of milk or other liquid, in which the fluid is brought into direct contact with the vibrating tube and is made to pass through areas where the sonic energy is highly concentrated. This process has been used successfully in the laboratory in the treatment of market milk. A more detailed report may be found elsewhere.[16]

### Heating Effects of Intense Audible Sound

When one's finger is placed under water near the center of the end-plate of the vibrating tube but not necessarily in contact with it, there is a very painful burning sensation. A similar sensation is felt when the tube is grasped firmly between the fingers near one end and vibrated in air at maximum intensity; that this burning sensation actually accompanies a decided increase in temperature of the flesh is proved by touching the fingers to one's cheek.

A cork firmly driven into one end of a nickel tube and vibrated in air or

or under water, quickly becomes so hot at its center, due to the degeneration of sound energy into heat within the viscous cork, that the gases of dry distillation blow out a vent in about 25 seconds. Corks into which the intense audible sound has been introduced and which have thus been "burned by sound" are shown in Fig. 12; each has been cut into halves with a razor blade to reveal the charred interior. The cork on the lower left was exposed to the sound for 2 minutes, the others for one minute each.

Fig. 12. Corks "burned by sound."

### Tactual Illusions

The following phenomena, while not heating effects, are closely associated with the above. A nickel tube held lightly between the fingers near one end and made to vibrate while the circuit is being tuned, seems to become very smooth as though covered by a film of oil. And, when the point of maximum intensity of vibration is being passed through, the tube seems to swell between the fingers. Test with calipers shows, of course, that no such enlargement is actually taking place. The phenomenon of apparent swelling proves very useful when it is desired that the tube vibrate in air at resonance

### Flocculation Produced by Audible Sound, Sonic Attractions. Radiation Pressure upon Submerged Bubbles

If the intense audible sound be introduced into a vessel containing water cloudy with tiny air bubbles, these immediately coalesce to form large bubbles which rise to the surface, the water becoming clear in a few seconds. A similar effect of ultrasonic sound of about 40 times the frequency has been reported by E. N. Harvey,[17] who ascribes the effect to radiation pressure. In

[17] Harvey, Biol. Bull. **59,** 306 (1930).

the case of audible sound, however, since the nodes of vibration in the water are far apart and the flocculation is observed to occur throughout the liquid, I am inclined to think that the major factor in bringing the bubbles together is sonic attraction due to the Bernoulli effect, which in the Kundt tube results in the apparent attraction of cork particles for each other.[18]

A stream of small air bubbles released under water near the end-plate of the vibrator is driven strongly away due to radiation pressure of the audible sound, provided the distance from the vibrator is greater than about one centimeter. If the bubbles be released within this distance of the end-plate, sonic attraction is sufficient to overcome both radiation pressure and force of buoyancy and the small bubbles move down to the vibrator plate.

Streams of bubbles released about $1\frac{1}{2}$ cm from the end-plate at various places above the level of its surface, indicate by their quick retreat from the end of the tube that the cone of strong radiation from this reciprocating plate occupies the full $2\pi$ steradians of the hemisphere, as would be expected from formula[19] the wave-length of the audible sound being large as compared with the radius of the piston-source, 17 cm as compared with 0.95 cm.

Radial vibration of the lateral surface accompanying longitudinal vibration of the nickel tube is indicated by repulsions and attractions of small air bubbles by the lateral surface in manner similar to the above but weaker.

## Bubbles Responsive to Sound

When a nickel rod is vibrating under water, bubbles of certain sizes ranging from about 2 mm in diameter downward, are to be seen within the water or against the sides of the vessel, dancing and darting about in a very erratic manner. Of particular interest is the fact that these bubbles, which I am calling "resonant" for lack of a better name, are translucent, the air-liquid surface being strongly disturbed causing them to appear white; and

Fig. 13. Path of a "resonant" bubble.

interesting also is the fact that, seemingly, only bubbles of certain sizes are thus erratic in their movements and translucent. The irregular lightning-like trace shown in Fig. 13 is a photograph of part of the path of one of these very active bubbles; dark field illumination was used, and one-fifth second exposure.

Resonant bubbles are often attracted one to the other, doubtless due to the Bernoulli effect, and here and there two or more such bubbles are seen to

[18] Koenig, Wied. Ann. **42,** 353 and 549 (1891).

[19] A. B. Wood, *A Textbook of Sound*, Macmillan (1930), p. 149.

join themselves together producing a single bubble which may possess neither the cloudy surface nor the erratic movements of its parents.

The translucent surface of a resonant bubble must be due to ripples of some sort, perhaps due to a ripple or ripple group that runs round and round bubbles of such circumference that the moving ripple gets an impulse each time a wave front passes the bubble. Two such systems of ripples running in opposite directions round a bubble, would produce standing waves on the air-liquid surface, and sudden changes in the position of the standing wave system conceivably might cause the erratic darting and dancing of the individual bubble.

As a rough check upon the reasonableness of the theory that a ripple or ripple group might run round and round a bubble getting an impulse each time a wave front passes the bubble, I shall derive an equation connecting the diameter $D$ of a resonant bubble with the number $n$ of successive ripples contained in the great circle of the bubble, $U$ denoting the velocity of the ripple and $f$ the frequency of the sound. It will be assumed for the sake of simplicity that $U$ is independent of $D$ and equal to the velocity of a ripple on the level surface of water, about 30 cm per second. A frequency of 8900 cycles will be used. Evidently $\pi D/n = U/f$.

To determine the smallest size of resonant bubble, i.e., the bubble for which a ripple makes one complete circuit between arrivals of successive wave fronts of the sound, solve for $D$ and let $n = 1$,

$$D = nU/\pi f = U/\pi f = 300/(\pi \times 8900) = 0.011 \text{ mm}.$$

It is observed that bubbles barely visible to the naked eye are often resonant; and so, there seems to be a degree of reasonableness to the above theory.

That the surface of a resonant bubble possesses something akin to circular motion is indicated by the following phenomenon. When a translucent bubble that has happened to attach itself to the side of the water vessel chances to capture a short bristle from foreign solid material in the water, the bristle will whirl around, sometimes clockwise sometimes counterclockwise.

A rapid, orbitual motion of a resonant bubble about some point on the side of the vibrating tube or on the side of the glass vessel is often observed, the bubble remaining in contact with the surface of the solid.

### "Atomization" of Liquids, a Possible Explanation

When the nickel tube is inverted and a drop of water or oil suspended from its closed end and the tube is vibrated, but with the circuit not tuned to exact resonance with the vibrator, the drop becomes translucent, the whitened appearance being due to the disturbed condition of the surface, Fig. 15. If, now, the vigor of oscillation is increased by tuning the circuit toward resonance, the cloudy drop ejects a fine spray from its tip. If, however, the circuit is suddenly tuned to the point of maximum response of the tube, the translucent drop ejects extremely fine drops of liquid from all parts of its surface.

Very similar "atomization" phenomena have been observed by Wood and Loomis[3] and other workers in the supersonic field.

In the present research, a file-like pattern is observed upon the surface of the vibrating drop, and it occurs to the author that the atomization is probably due in part to this ripple pattern's becoming so pronounced as to break into the fine spray. A similar phenomenon occurs when the edge of a bowl containing water is strongly bowed.[20] It is also probable that cavitation within the drop aids in the formation of the spray.

If such a ripple pattern occurs at the interface of liquids when exposed to sound. it may offer mechanism for the emulsification observed when such boundaries are exposed to very intense sounds.

Fig. 14. Drop of water hanging from stationary end-plate of nickel tube.

Fig. 15. Same as Fig. 14, but with tube vibrating longitudinally.

## Sound Reduces Pressure Within an Oil Drop

If the nickel tube be inverted and its closed end dipped into water or oil and then raised, a much larger drop of liquid remains attached to it when it is vibrating than when it is not. If the vibration is made more intense, the drop is seen to flatten out against the end-plate. This flattening is clearly shown in Figs. 14 and 15. In the first figure the tube is stationary and a drop of ordinary shape and size is suspended from its end. In Fig. 15, the rod is vibrating, the drop has changed much in shape and seems to be pushed up against the vibrator.

The Bernoulli principle is the basis of this effect. Evidently some of the sound within the drop is moving parallel to the end-plate, the vibration of the particles of liquid causes a reduction of the pressure within the drop, so that the atmosphere presses the drop up against the end of the rod. The phenom-

[20] Bragg, *The World of Sound*, G. Bell and Sons, Ltd., London, p. 88.

enon is thus similar to an effect observed by Guthrie,[21] that a delicately suspended rectangle of paper is attracted by a vibrating tuning fork brought close to it, closeness of fork to the paper surface being essential to the action, and the phenomenon is akin to the attractions between bubbles and between bubble and vibrator, mentioned above in this paper.

To test the close kinship of the phenomenon described to that of Guthrie, the writer dipped an ordinary tuning fork into liquid and readily obtained flattening out of drops of liquid against the vibrating prongs of the fork. He also found that the end-plate of the vibrating nickel tube shows decided attraction for bits of paper brought close to it, indeed, that a small disk of paper attracted up to the end-plate remains "sticking" to it as long as the tube vibrates.

The author wishes to call attention to the fact that a drop of liquid suspended from the moving end-plate is itself a source of sound and, since this sound exerts pressure upon surrounding objects, a force of reaction of radiation pressure must be acting upon the drop itself, which force must to some extent be responsible for the flattening of the drop against the end-plate of the vibrator. Experimental separation of the effect of force due to back pressure of radiation from that due to the Guthrie effect seems difficult.

### Nickel Tube Vibrating at Two Unrelated Frequencies at Once

A nickel tube vibrating in air, when actuated by currents of nearly the natural frequency of the tube, is often observed to emit beats, which probably are due to the tube's vibrating at the frequency of the circuit and at its own natural frequency at the same time. By carefully varying the capacity, $C_1$, the operator can reduce the number of beats as low as four or five per second before tube and circuit pull into step with each other.

### A Mode of Vibration Half the Frequency of the Fundamental

When a magnetostrictive vibrator is rested upon a hard body and operated in air at full amplitude of vibration, the tube when warm often gives, in addition to the fundamental and overtones, a strong musical tone having half the frequency of the fundamental. Several tubes, of diameters up to 10 cm and of different lengths, have thus been made to give this lower tone.

The production of a "sub-harmonic" by vibrating bodies,—bells, tuning forks, and piezoelectric crystals, when in contact with a stationary body, —has been reported by several investigators,[22] but never, so far as the writer knows, in the case of magnetostrictive vibration.

### Kundt Tube Striations One Particle Thick

As might be expected, the vibrator when used in air in a horizontal position proves to be a very effective means of exciting a Kundt tube, very tall striations of cork dust being obtained.

[21] A. B. Wood, reference 19, p. 434. Also, Rayleigh, *Theory of Sound*, Vol. 2, p. 43.
[22] A. B. Wood, reference 19, p. 168.

When half grains of "puffed rice" are used instead of cork dust, the sound is sufficient to raise low striation walls. As demanded by Koenig's striation theory,[18] these walls are everywhere but one particle thick.

### Longitudinal Striations Produced in a Kundt Tube

When a strongly vibrating nickel tube is introduced into a glass Kundt tube of considerably larger diameter than the vibrator, it is found possible to produce striations of cork dust running *parallel* to the axis of the Kundt tube rather than perpendicular to it, also to form scallops running the length of the tube. These scallops are possibly a combination of the longitudinal striations and the ordinary transverse type.

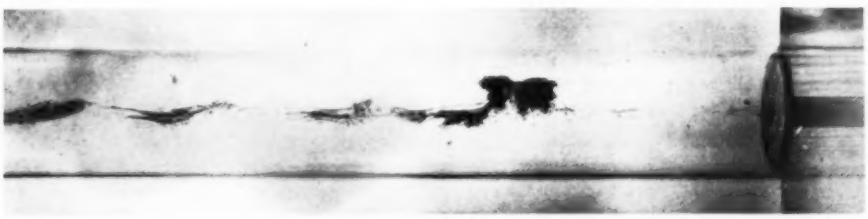

Fig. 16. Longitudinal striations produced in Kundt tube, top view.

Fig. 16 shows striations parallel to the axis of the glass tube, which is of 4.4 cm internal diameter. The reflector is a brass disk slightly less in diameter. The diameter of the vibrator and of its end-plate is 1.9 cm (3/4 inch). In order to obtain the longitudinal type of striation, it is found necessary that the axis of the vibrator make an angle with the axis of the glass tube.

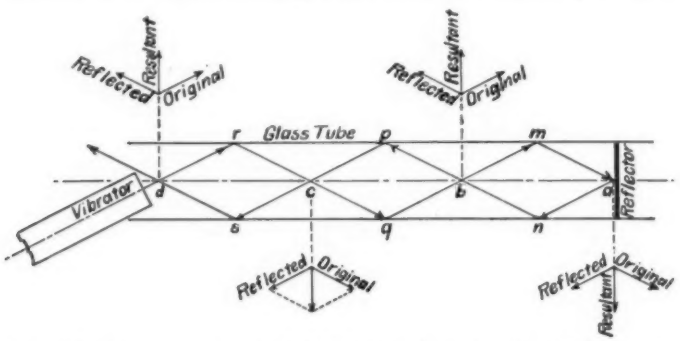


Fig. 17. Diagram to explain formation of longitudinal striations.

Now, according to Koenig's theory,[18] the striations must form at right angles to the direction of propagation of the sound. Most of the sound, then, in Fig. 16 must be moving at right angles to the axis of the glass tube, probably forming standing waves between opposite walls of the tube.

As to the manner in which sound of such direction could be produced, consider a wave front that starts from the end-plate of the vibrator, Fig. 17, and is reflected from one wall to the other until it strikes the brass disk; here

it is regularly reflected and makes a zigzag return, the return path being symmetrical with the other, the axis of the glass tube being axis of symmetry. The wave reflected at the brass disk will cross the oncoming wave at places *a*, *b*, *c*, *d*, etc. If paths *b-m-a-n-b*, *c-p-b-q-c*, etc. are a whole number of wavelengths, the motion of air particles at *a*, *b*, *c*, *d*, etc. will be at right angles to the axis of the Kundt tube, the motion of the air particles at the particular instant shown in Fig. 17 being toward one side of the glass tube at places *a*, *c*, etc. and toward the other side at places *b*, *d*, etc. The directions of the resultant sound at the places under consideration would be, of course, only approximately at right angles to the axis of the Kundt tube, because the vector of the returning reflected sound is weaker than that of the oncoming original sound because of losses due to absorption, diffraction, etc.

The theory just given is strongly supported by two facts. First, if longitudinal striations are found in the tube at all, there are always striations of this type *next* to the reflector. Second, when, as mentioned in the preceding paragraph, the striations take the form of scallops running lengthwise of the tube, it is observed that these scallops curve first to one side of the Kundt tube then to the other, suggesting that the resulting motion of air particles has opposite direction at adjacent scallops. The bowing of the longitudinal striation is probably partly due to variation of the phase of the reflected wave which would result in a slight difference in the direction of motion of air particles located to left and right of each of the points *a*, *b*, *c*, etc. A slight tendency toward the scalloped form is shown by the striations illustrated in Fig. 16.

In conclusion, the writer wishes to express his sincere thanks to Dr. S. Leroy Brown, under whose direction the research was undertaken and who suggested the particular field, to Dr. C. P. Boner for advice in connection with assembly of the apparatus, and to Mr. Louis H. Gruber, the mechanician.

# Three Superfluous Systems of Electromagnetic Units[1]

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At the present time the electromagnetic, electrostatic, Heaviside-Lorentz, practical and international systems of electric and magnetic units are used side by side in pure and applied electromagnetism. The question is here raised whether the use of this multiplicity of units should continue indefinitely into the future when the conversion tables for translating from any system to any other system show the essential equivalence of all five systems. It is recommended that but one system be legalized and used generally in place of the five systems, and that this universal system be the coherent meter-kilogram-second-ohm or definitive system. It is further recommended that the international ohm be used in this system. This unit is the one actually used in exploring the physical world because laboratory resistances for physics and test room resistances for engineering have been so calibrated. Of far greater importance is the fact that by retaining the international ohm it will be simpler, and completely feasible, to eliminate what Heaviside called "that unmitigated nuisance, the $4\pi$ factor of the present B.A. units" from our preferred system of units.

## I. INTRODUCTION

IN THE invitation to contribute to this *Symposium on Electric and Magnetic Units*, special attention was called to assigning unit magnitudes in the practical magnetic system. I favor the system based on the ampere-turn, volt-second and meter, all units to be supplied with the most convenient names. In discussing magnetic units, it is most desirable to consider the electric, magnetic and mechanical units comprehensively. I suggest starting the discussion by considering a system of units which has been enthusiastically recommended in Italy, England and Germany, as well as in our own country. It is the meter-kilogram-second-ohm system, proposed in 1901 by G. Giorgi and independently in 1904 by David Robertson. Endorsing the logical reasoning of earlier writers, the proposal is here made that the m.k s.Ω. system be used for all purposes. It is shown that positive advantages would inevitably follow. At the present time, scientific and engineering papers by the hundred could obviously be cited which already make exclusive use of m.k.s.Ω. units. I refer to papers using no units except the thirty-six units listed in Table II and briefly discussed below, together with all decimal multiples of these units obtained by adding the usual decimal prefixes.

The seventy-year domination of the so-called absolute cgs electromagnetic units is shown to rest on a naive faith. The established authority for the cgs tradition is questioned. It is hoped that this discussion will aid in overcoming the general feeling that there is something very sacred about the

[1] A paper written for the *Symposium on Electric and Magnetic Units* at the Physical Society Meeting, Washington, D. C., April 30, 1932, and somewhat expanded within the framework of a ten minute paper.

cgs system. It is proposed to retain and stabilize the international ohm at its present value. A difference of one-twentieth of one percent thus becomes a touchstone. This would be only a small percentage change, but making it would prolong the specious authority of the cgs system, perpetuate the "swarm of $4\pi$'s" and delay realization of the full benefits derivable from the m.k.s. Ω. system.

## II. Retain the International Ohm

I propose that the international ohm be used as the unit of electrical resistance for all purposes, both practical and purely scientific. Thus, it would remain unchanged in magnitude as the legalized ohm and, in addition, it would supersede all of the electromagnetic and electrostatic units of resistance. This is a counter proposal to the resolutions adopted during the past five years by various advisory committees[2] which have been looking definitely toward international legalization establishing electrical units based upon the value $4\pi$ for the inductivity of free space. These resolutions support the International Committee on Weights and Measures in their tentative 1927 decision to abandon the international ohm in the near future. Units should now be considered very definitely with a view to their use over a period of fifty years because revised laws, defining new international electric and magnetic units, are now the real subject of discussion.

My proposal aims to put an end to the duality in electrical units which has been responsible, during the past seventy years, for much confusion, waste of time and inadequate mental grasp of physical magnitudes. The other proposal would continue the enormous gap, represented by the factor $10^9$, between the ohm of the test room and the electromagnetic unit of resistance of pure science.

My proposal would make the inductivity of free space an experimental constant. The other proposal continues the arbitrary, implicit choice of $4\pi$ for this constant (as explained in the note to Table I) which would perpetuate the eruption of $4\pi$'s in electromagnetic equations, with all its troublesome consequences.

In addition to the above two permanently continuing disadvantages of the proposal to abandon the international ohm, there would be transition disadvantages. My proposal would call for no change in the material resistances, inductances and capacities which are now in use. The proposal of the $4\pi$ adherents would tend to make obsolete all refined apparatus in these categories, adjusted to one-twentieth of one percent, and this would impose a financial burden upon many institutions. A manufacturer's guarantee of one-twentieth of one percent for the absolute value of resistance apparatus would fall to a guarantee of one-tenth of one percent when the unit of resistance was changed by one-twentieth of one percent.

My proposal ties the resistance unit as closely as possible to the present wire coils of the several national bureaus, and requisitions every means for

[2] Tech. News Bull. Bur. Standards No. 135, 93–96 (1928); No. 164, 115–116 (1930); No. 171, 74 (1931).

stabilizing these material standards from time to time in the most rigorous manner possible, at the time, in the hope of ultimately reaching a prototype ohm,[3] ranking with the prototype meter and the prototype kilogram. The proposal of the $4\pi$ adherents follows the mirage which lured on the British Association Committee on Standards of Electrical Resistance in spite of the endless pursuit which they foresaw, and their opinion that "few defects could be more prejudicial than this continual shifting of the standard" material ohm.

## III. The Arbitrariness of the Absolute Unit of Resistance

The $4\pi$ adherents in their proposals refer to the "absolute," "true" or "fundamental" ohm as though it were a cornerstone of the universe. The facts seem to be quite different. The absolute magnitudes of the absolute units were chosen arbitrarily. After the plan of duality for electrical units, utilizing both a material unit and an absolute unit, had been proposed, explained and adopted in the first report of the British Association Committee, that report continues: "An infinite number of systems might fulfill the above conditions which leave the absolute magnitude of the units undetermined. Weber has proposed to fix the series in various ways, of which two only need to be mentioned here." Thus, out of an unlimited range of possibilities, the British Association Committee limited themselves to mentioning two. At the time no one seems to have raised the question as to whether there might have been a more fundamental or more convenient magnitude among the other systems considered by Weber, or among the infinite number of systems which no one had considered up to that time. In particular, no one seems to have asked the specific question as to why the magnitude of the absolute unit of resistance should not be chosen identical with that of the material standards of resistance, which had been roughly fixed upon earlier in the same report.

If the British Association Committee had looked into the question with reasonable care in 1862, they would have found that the Siemens meter of mercury would have been a most convenient unit for all purposes. The world would then have been spared the B.A. ohm, the legal ohm and the international ohm, as well as all the present absolute units of resistance. If the Siemens unit had been adopted in 1862 for all purposes, both practical and purely scientific, there could not have been the confusion and uncertainty which has existed during the past seventy years. The permanence of material

[3] Mr. W. J. Shackelton believes that greater stability can be realized in an inductance coil than in a resistance coil. Until it is established whether some one material electrical standard has the desired stability, experimental investigations should correlate all types of electrical standards. This would call for the direct determination not only of the ohm and ampere, but also of the volt, henry, farad and coulomb. Such determinations made at all of the national bureaus should be critically cross checked and weighted to determine the most probable averages and the maximum possible errors. Of course, this involves great experimental difficulties and a balanced skeptical objectivity on the part of all the scientists involved. Care should be taken that legalizing acts allow every advantage to be taken of advances in the science of measurements.

standards of resistance would have been studied much more broadly and intensively if the implications had been recognized of the fact that, while the scientific experimenter may go to endless inconvenience for the sake of greater precision, he can never experiment with a unit "which has no material existence, but is rather an abstraction than an entity." The actual history of the ohm has been given in this summary: "The electrical unit for which an accurate value is most needed, on which the most effort has been expended, and for which the results have been most unsatisfactory, is the ohm."[4]

If, on the other hand, they had pursued the subject in a different direction, they would have found that the characteristic impedance of electromagnetic waves in free space is a fundamental resistance of the universe. In 1864 Maxwell read his paper, *A Dynamical Theory of the Electromagnetic Field*, which brought in the velocity and characteristic impedance of light, explicitly and implicitly, respectively. The characteristic impedance of free space is defined to be the quotient of the square of the effective potential gradient in volts per meter, for a plane electromagnetic wave, divided by the transmitted watts per square meter. It also equals the square root of the quotient of the inductivity in henries per meter divided by the capacitivity in farads per meter. The characteristic impedance is thus an inherent electrical resistance presented by free space which is fundamentally associated with the velocity of light. Maxwell's numerical data for sunlight (as recalculated in his Treatise, II, p. 441, footnote) corresponds to 666.1 effective volts per meter, 1.176 kilowatts per square meter, and 377.3 ohms for the characteristic resistance or impedance of free space.

The British Association Committee finally adopted both Weber's electromagnetic and electrostatic absolute units of resistance, after multiplying each of them by ten. Taken with the practical units which they also advocated, this makes three systems. Sanctioning this multiplicity of electrical units may have been due, in part, to an accidental oversight. The committee decided that it was important to repeat Weber's experiments with the greatest possible refinement. In this connection they also adopted his terminology without perhaps fully considering that his terminology did not necessarily go with his experiments.

## IV. Stabilization of the International Ohm

My proposal calls for additional precise determinations of the constants of free space. The resolution of the $4\pi$ adherents calls for additional absolute electrical measurements. Both plans call, however, for identically the same information about the physical world, obtainable with identically the same apparatus. The difference between the two plans is thus essentially a matter of words. I would postulate the international ohm and say that a Lorentz apparatus measurement, for example, gives the inductivity of free space. The $4\pi$ adherents postulate the arbitrary value $4\pi$ for the inductivity of free space and say that a Lorentz apparatus measurement gives the absolute

[4] Harvey L. Curtis, *The Determination of the Electrical Units by Mechanical Measurements*, J. Wash. Acad. Sci. **22**, No. 8, 201 (1932).

value of a resistance. Different words are used but the same point has been reached in our descriptions of the actual physical world.

Table I shows four constants of free space, all of which I would leave for experimental determination—due attention being given to the two connections between them mentioned in a footnote to the table. The constants are roughly: inductivity, 1.256 microhenries per meter; capacitivity, 8.86 micro-microfarads per meter; velocity, 300 megameters per second; characteristic impedance, 377 ohms. Stabilizing the international ohm to correspond with 1.256 microhenries per meter for the inductivity of free space calls for identically the same experimental researches and can, therefore, be carried out with identically the same precision as if the value $4\pi/10$ had been chosen.

TABLE I. *Constants of free space expressed in five systems of units.*

| Constant of free space** | Systems of units with their postulated primary units | | | | |
|---|---|---|---|---|---|
| | Definitive meter, watt, second, international ohm | Practical $10^9$ centimeter, watt, second* | Electromagnetic cgs* | Electrostatic cgs* | Heaviside-Lorentz cgs* |
| Inductivity $\nu$ | 1.25600 microhenries per meter | $4\pi =$ 12.566371 | $4\pi =$ 12.566371 | $4\pi c^{-2} =$ $1398.16\times10^{-13}$ | $c^{-2} =$ $1112.62\times10^{-24}$ |
| Capacitivity $\kappa$ | 8.85849 micro-micro-farads per meter | $(4\pi c^2)^{-1}10^{18} =$ $88.5398\times10^{-4}$ | $(4\pi c^2)^{-1} =$ $88.5398\times10^{-24}$ | $(4\pi)^{-1} =$ $79577.47\times10^{-6}$ | 1 |
| Velocity $v$ | 299.796 megameters per second | $c\ 10^{-9} =$ 29.9796 | $c =$ $29979.6\times10^{5}$ | $c =$ $29979.6\times10^{5}$ | $c =$ $29979.6\times10^{5}$ |
| Characteristic impedance $K$ | 376.543 $(1+i0)$ ohms | $4\pi c\ 10^{-9} =$ 376.735 | $4\pi c =$ $0.376735\times10^{12}$ | $4\pi c^{-1} =$ $41.9164\times10^{-6}$ | $c^{-1} =$ $33.3560\times10^{-12}$ |

*Note:* The table is based on the accepted values 299.796 megameters per second for the velocity of light and 1.00051 absolute ohms for the international ohm.

Inductivity $\nu$ and capacitivity $\kappa$ are defined uniformly with conductivity $\gamma$ so that the formulas $\mathcal{L}=\nu SL^{-1}$, $C=\kappa SL^{-1}$ and $G=\gamma SL^{-1}$ give the inductance, capacity and conductance, respectively, due to fluxes in right cylinders of cross section $S$ and length $L$, provided the coil winding has but one turn and all fluxes are uniform and parallel to the axes of the cylinders.

* The fourth unit in this system is supplied by the implicitly postulated constant 1 or $(4\pi)^{\pm1}$ for the inductivity or capacitivity.

** The four constants are connected by the two relations $\nu vK^{-1}=\kappa vK=1$.

When the experimental determinations of the constants of free space can be made to one part in a million, they may be of controlling importance in stabilizing the international ohm at its present value. This would be analogous to the stabilization of the prototype meter by means of the wavelength of light. At present, it is my understanding, however, that the precisions attainable with the Lorentz apparatus and the international ohm are about on a parity. The actual maximum differences from the mean for the international ohm are about ten parts and for the absolute ohm about twenty parts in a million.[5]

The exact value at which the international ohm should be stabilized might well be the mean of the wire resistance standards in these national bureaus. Its value would thus be fixed, apparently, within a few parts in a

[5] These results, reported in 1931, are for the most recent published comparisons of the British, German and U. S. modern wire resistance secondary standards and for the absolute measurements of the same bureaus. Tech. News Bull. Bur. Stand., p. 75, July, 1931; H. L. Curtis, Trans. A.I.E.E., **50**, 1325–6 (1931).

million. It is agreed that there is no occasion for reviving the mercury standard ohm for any further measurements.

For the $4\pi$ adherents the basic constants of free space are given in the column of the table headed *Electromagnetic*. The velocity depends upon experiment, while the value of the inductivity is $4\pi$ in consequence of the postulates. The inductivity of free space in this system is thus known to 700 decimal places. This mathematical fact is, of course, of no significance in connection with the external physical world. When any abstract unit, such as the electromagnetic unit of resistance, enters the physical laboratory for actual physical measurements, it must be given a material embodiment. As soon as this is done, it acquires the serious limitations which are inherent in all material standards. Thus, the Lorentz apparatus of the National Physical Laboratory, which is a massive material entity including 200 kilograms of marble as an essential part, has its own temperature coefficient.

It follows that the cgs electromagnetic system of units is superfluous because: it adds nothing to our knowledge of the physical world; it makes no contribution towards stabilizing the magnitude of the legalized material ohm; its terminology is inferior to that of the units which are actually used in making physical experiments. It should be abolished. The same conclusion applies to the cgs electrostatic system, to the Heaviside-Lorentz system and even to the practical system as derived from the cgs system. It will be seen by reference to Table I that both the practical and electromagnetic systems implicitly assign the inductivity $4\pi$ to free space, while the electrostatic and Heaviside-Lorentz systems assign values $(4\pi)^{-1}$ and unity, respectively, to the capacitivity of free space.

One system is sufficient for universal use. It is far better that this system should not endow the constants of free space with integral or other familiar magnitudes. Assigning the value unity to all four constants of free space would be interesting but not practically desirable. An arbitrary unit of resistance, such as the international ohm, is preferable.

Retaining the international ohm, as proposed, does not merely shift the $4\pi$. The resulting inductivity of free space, ignoring the decimal point, still has practically the value $4\pi$ because it differs therefrom by only one-twentieth of one percent. It would confessedly be better if the discrepancy were greater. But it hardly seems worth while deliberately introducing a change from the magnitude of the international ohm for this reason alone. When the inductivity of free space is regarded as an experimental constant, the $4\pi$ tradition can be lived down. Other experimental constants fall near familiar numerical magnitudes. Thus, the velocity of light, again ignoring the decimal point, happens to differ from the integer 3 by less than one-fifteenth of one percent. If the rising generation is taught that the ohm in actual use was finally assigned an arbitrary magnitude and that the inductivity, capacitivity and impedance of free space are thus experimental constants, just as is the velocity of light, they will have a new point of view about electric and magnetic units.

## V. Completing the Definitive System of Units

Assuming that we have settled upon the international ohm as the initial electrical unit of our system, it is logical to take the mechanical watt, and not the ampere, as the next unit, because power is the great connecting link between all physical measurements. The ampere and volt are thereby fixed in magnitude. Adding the second of time as a unit of the system then fixes the size of the henry, farad, coulomb and joule. The unit of length is still optional. The meter seems to present preponderant advantages; the coherent unit of mass is the kilogram. The name "kram" was substituted by Robertson to eliminate the decimal prefix. Six of these eleven units are international units; this leaves the ampere, volt, coulomb, watt and joule, which differ from the international units by one-thirtieth of one percent or less. Derived units would include the meter per second, volt per meter, henry per meter, farad per meter, meter-ohm and ampere per square meter.

Having avoided thus far the troublesome $4\pi$ of the electromagnetic system, we would certainly not voluntarily introduce it in connection with the magnetic units. They should be based on the ampere-turn, the volt-second, the ampere-turn per meter and the volt-second per square meter.

In terms of these electric and magnetic units, the fundamental equations for free space take on the form:

$$\operatorname{curl} \mathbf{H} = \kappa \dot{\mathbf{E}},$$
$$-\operatorname{curl} \mathbf{E} = \nu \dot{\mathbf{H}},$$

which leave nothing to be desired. For media in general the fundamental equations also have simple, symmetrical forms.[6]

The unit names for a system are of great practical importance. These names should not be derived from the unit names in other systems by adding a prefix, such as "pra" to form "pra-maxwell," nor should they include decimal prefixes, which should be used only to indicate multiples and submultiples of the primary or reference units. This elementary rule was disregarded in the centimeter of the cgs systems (over the written protest of G. Johnstone Stoney) and constitutes one of the many defects of these systems. The poverty of unit names in the three cgs systems, while consistent with the esoteric ideals underlying them, is another handicap to their general use.

[6] The fundamental curl, wave and activity equations are written:

$$\operatorname{curl} \mathbf{H} = \gamma \mathbf{E} + \kappa \dot{\mathbf{E}} = (\Gamma/K)\mathbf{E};$$
$$-\operatorname{curl} \mathbf{E} = \xi \mathbf{H} + \nu \dot{\mathbf{H}} = (\Gamma K)\mathbf{H};$$
$$\Delta \mathbf{E} = \nu\kappa \ddot{\mathbf{E}} + (\xi\kappa + \nu\gamma)\dot{\mathbf{E}} + \xi\gamma \mathbf{E} = \Gamma^2 \mathbf{E};$$
$$-\operatorname{div} (\mathbf{E} \times \mathbf{H}) = (\gamma \mathbf{E} + \kappa \dot{\mathbf{E}}) \cdot \mathbf{E} + (\xi \mathbf{H} + \nu \dot{\mathbf{H}}) \cdot \mathbf{H} = (\Gamma/K)\mathbf{E}^2 + (\Gamma K)\mathbf{H}^2;$$
$$\Gamma = [(\xi + i\omega\nu)(\gamma + i\omega\kappa)]^{1/2} = \text{propagation constant};$$
$$K = [(\xi + i\omega\nu)/(\gamma + i\omega\kappa)]^{1/2} = \text{characteristic impedance};$$

where $\gamma$, $\kappa$, $\xi$ and $\nu$ are the conductivity, capacitivity, magnetic conductivity (Heaviside) and inductivity of the medium. The discussion of any medium is thus made formally uniform with that of any network; $\gamma$, $\kappa$, $\xi$ and $\nu$ for the medium correspond to G, C. R and L for the network.

Robertson published his proposal under the title *The Completion of the Practical System of Units*,[7] and he wished to have his system regarded as "merely an extension of the practical system to other quantities for which it had not hitherto been employed." It is necessary, however, to make a number of choices among the old units and the old names which are regarded as practical at the present time. These choices should be perfectly definite. This suggests the name "definitive system" which seems to present distinct advantages over "absolute practical system" proposed by Giorgi.

TABLE II. *Definitive units, symbols, defining equations, and conversion factors.*

| | Quantity | Name of definitive unit | Magnitude symbol and defining equation | Value of cgs electro-magnetic unit |
|---|---|---|---|---|
| 1 | length | meter | $L$ | $10^{-2}$ |
| 2 | area | square meter | $S=L_1L_2$ | $10^{-4}$ |
| 3 | volume | stere | $V=LS$ | $10^{-6}$ |
| 4 | time | second | $T$ | 1 |
| 5 | frequency | cycle per second | $f=NT^{-1}$ | 1 |
| 6 | velocity | meter per second | $v=LT^{-1}$ | $10^{-2}$ |
| 7 | acceleration | meter per second per second | $a=vT^{-1}$ | $10^{-2}$ |
| 8 | power | watt | $P$ | $10^{-7}$ |
| 9 | energy | joule | $W=PT$ | $10^{-7}$ |
| 10 | action | joule-second | $A=WT$ | $10^{-7}$ |
| 11 | force | (deci-megadyne) | $F=WL^{-1}$ | $10^{-5}$ |
| 12 | pressure | (deci-megadyne) per square meter | $p=FS^{-1}$ | $10^{-1}$ |
| 13 | mass | (kilogram) | $M=Fa^{-1}$ | $10^{-3}$ |
| 14 | momentum | (kilogram)-meter per second | $\mathbf{M}=Mv$ | $10^{-5}$ |
| 15 | density | (kilogram) per stere | $d=MV^{-1}$ | $10^{3}$ |
| 16 | torque | (deci-megadyne)-meter | $\tau=FL$ | $10^{-7}$ |
| 17 | electricity | coulomb | $Q$ | $\zeta 10$ |
| 18 | electric surface density | coulomb per square meter | $\sigma=QS^{-1}$ | $\zeta 10^{5}$ |
| 19 | electric current | ampere | $I=QT^{-1}$ | $\zeta 10$ |
| 20 | current density | ampere per square meter | $\mathbf{i}=IS^{-1}$ | $\zeta 10^{5}$ |
| 21 | electric potential | volt | $E=WQ^{-1}$ | $\zeta^{-1}10^{-8}$ |
| 22 | electric field | volt per meter | $\mathbf{E}=EL^{-1}$ | $\zeta^{-1}10^{-6}$ |
| 23 | electric capacity | farad | $C=QE^{-1}$ | $\zeta^{2}10^{-9}$ |
| 24 | capacitivity | farad per meter | $\kappa=CL^{-1}$ | $\zeta^{2}10^{11}$ |
| 25 | electric resistance | ohm | $R=EI^{-1}$ | $\zeta^{-2}10^{-9}$ |
| 26 | resistivity | meter-ohm | $\rho=RL$ | $\zeta^{-2}10^{-11}$ |
| 27 | electric conductance | mho | $G=IE^{-1}$ | $\zeta^{2}10^{9}$ |
| 28 | conductivity | mho per meter | $\gamma=GL^{-1}$ | $\zeta^{2}10^{11}$ |
| 29 | electric inductance | henry | $\mathfrak{L}=RT$ | $\zeta^{-2}10^{-9}$ |
| 30 | inductivity | henry per meter | $\nu=\mathfrak{L}L^{-1}$ | $\zeta^{-2}10^{-7}$ |
| 31 | magnetomotive force | ampere-turn | $\mathfrak{F}=NI$ | $\zeta(4\pi)^{-1}10$ |
| 32 | magnetic field | ampere-turn per meter | $\mathbf{H}=\mathfrak{F}L^{-1}$ | $\zeta(4\pi)^{-1}10^{3}$ |
| 33 | magnetic flux | volt-second | $\Phi=ET$ | $\zeta^{-1}10^{-8}$ |
| 34 | magnetic flux density | volt-second per square meter | $\mathbf{B}=\Phi S^{-1}$ | $\zeta^{-1}10^{-4}$ |
| 35 | magnetic reluctance | ampere-turn per volt-second | $\mathcal{R}=\mathfrak{F}\Phi^{-1}$ | $\zeta^{2}(4\pi)^{-1}10^{9}$ |
| 36 | magnetic permeance | volt-second per ampere-turn | $\mathcal{P}=\Phi\mathfrak{F}^{-1}$ | $\zeta^{-2}4\pi 10^{-9}$ |

$\zeta^2=(1.00025)^2=1.00051=$ ratio international ohm to "absolute" ohm.

Table II has been added to this paper to prevent the possibility of the feeling arising on the part of the reader that there is anything essentially novel, intangible or impractical about the complete system of definitive units.

[7] David Robertson, The Electrician, pp. 24–25, April 22, 1904; p. 240, May 27, 1904.

In this table, thirty-six of the more frequently used units are listed in the mechanical, electrical and magnetic classes. The defining equations are simply the product or the quotient of two units occurring at an earlier point in the same table. The only exceptions are for the meter, second, watt and coulomb which were arbitrarily chosen as the basic four units because they are of great physical importance and they are easily visualized. Their choice for this particular purpose in this particular table is in no way in conflict with the choice of any other units for the precise maintenance of the entire definitive system by standardizing bureaus. It is expected that new names would be assigned to a few units, such as the (deci-megadyne) and (kilogram), which are enclosed in parentheses.

## VI. Use and Legalize Definitive Units

Fortunately, there is no inherent difficulty involved in initiating the exclusive use of definitive units. Every member of the Physical Society already uses many of these units and he may introduce the other units of this system into his papers and into his teaching without difficulty or fear of misunderstanding. The exclusive use of a single system of units, made familiar by practical use in the home and test room, is most desirable. It seems incredible that students should still be required to learn a multiplicity of electrical units when, aside from references to the literature, they will be better off with but one system of units for both their practical and theoretical work, either as engineers or as scientists. Textbooks should define definitive units without making any reference to cgs units. It has been quite true that "the entire system of units used in electrical measurements is a monument to the ingenuity of science, but productive of many difficulties to students." By removing this monument from the textbooks to the histories, the student's path can be materially smoothed and shortened.

When, in 1933, the International Bureau of Weights and Measures assumes responsibility for electrical standards, they can adopt for the ohm a definite magnitude which satisfactorily represents the international ohm now in actual use. This would correspond to their adoption, in 1875, of the magnitudes which were then in actual use for the meter and kilogram. For reasons explained above, I believe that the arguments in the case of the ohm are as plain and convincing as they were in the case of the meter and the kilogram. To change the magnitude of the ohm would not only be futile, but it would go a long way toward making permanent both the eruption of $4\pi$'s and the multiplicity of electrical units.

Presumably ninety-nine percent of the members of the Physical Society have no call to devote valuable time to the revision of units; they are interested primarily in using units as tools. Nevertheless, I believe that for the common interest the members of this society should actively advocate the international adoption of the definitive system of units, and that they should oppose legalization of the cgs system in the magnetic class or in any other way. It will be simpler to prevent the legalization of any cgs units now, when the first proposals are being made, than it will be to abandon these cgs units

if they once become established by legal enactments. Internationally legalized units should all form a coherent system and not a mixed jumble of units. All new legalized units should constitute a coherent extension of the electrical units which are now legalized. When the U. S. Act, approved July 12, 1894, is revised, it should be made to define strictly coherent definitive units, and the four references to the cgs system should be deleted, since they were, of course, always out of place. The act should continue to leave complete scientific freedom as regards the use of cgs and other units. Such freedom will always be made use of, to a limited extent, in some special fields. This should not interfere with the early, practical realization of the ideal.

"The goal to be hoped for is a single, universal, comprehensive, scientific system of units for theory, practice, and international development in all scientific branches, everywhere. The only avenue in sight for reaching this goal seems to be through the m.k.s. Ω. system."[8]

## VII. Summary

It is indeed surprising that the use of our mechanical, electric and magnetic units can be so much simplified by the introduction of a few minor, conservative, well considered changes. A concise list of the simplifications, together with the related changes, follows:

1. Uniformity in units for all practical and theoretical work is made possible by taking the m k.s.Ω. system as the normal, preferred system.

2. That unmitigated $4\pi$ nuisance is ended and simple physical equations secured by adhering to the arbitrary international ohm and the ampere-turn.

3. Magnetic units are made another aspect of electric units by adhering to the ampere-turn and the volt-second.

4. Unit names are made most helpful by a few changes required by sound principles of nomenclature.

6. Recalibration of precision apparatus is minimized by adhering to the international ohm, henry and farad.

[8] A. E. Kennelly, *Historical Outline of the Electrical Units*, Jour. Eng. Ed. **XIX**, [3] 271 (1928).

# A Vacuum Apparatus for Measuring Thermal Expansion at Elevated Temperatures, with Measurements on Platinum, Gold, Magnesium and Zinc

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An apparatus for measuring linear thermal expansion between room temperature and 950°C in vacuo by an interference method has been developed and tested on platinum, gold, magnesium and single-crystal zinc. Although designed primarily for metals which oxidize readily it seems to give excellent results for all metals in the temperature ranges through which their vapor pressure is low. Under conditions such that a metal has a high vapor pressure the performance is not quite so satisfactory because of the deposition of metallic films on some of the cooler parts of the optical train. The refractive index of fused silica for the helium line $\lambda = 5877.2$ has been calculated for the temperature range 18° to 950°C. A study of the dilatation of single-crystal zinc yields no evidence for the existence of more than one form between 20° and 350°C.

## INTRODUCTION

ONE of the most precise methods at present available for measuring linear thermal expansion is the interferometric method devised by Fizeau in the middle of the last century. With the ever-increasing demand for a more accurate knowledge of the properties of matter, particularly at high temperatures, the principle on which the method is based has found increasing favor and the original apparatus has passed through a number of modifications[1] in its adaptation to various uses.

The advantages of the scheme are briefly as follows. (1) The change in length can be determined with a precision of the order of magnitude of half a wave-length of light. (2) This precision makes it possible to work with small specimens (5 mm or less) which simplifies the problem of temperature distribution to such an extent that a uniformity of temperature can be obtained which is rarely achieved by any other method. (3) The specimens can be irregular in shape, a feature of great value in dealing with very hard materials. (4) The apparatus, having no moving mechanical parts, possesses an unusual simplicity of construction. (5) A great number of observations can be made during any single test. Expansion data covering the temperature range 20° to 1000°C and based on as many points as the operator desires are easily obtained in a single day. This ability to follow the expansion more or less continuously is of particular value in studying the behavior of a substance in passing through an inversion point; it also enables one to discover small irregularities in expansion which might otherwise escape notice.

[1] For a review of the chief modifications see Valentiner, Wien-Harms Handb. d. Exp. Physik **VIII** 2nd part, p. 3 or Grüneisen, Geiger-Scheel. Handb. d. Physik **X**, p. 23.

Along with these advantages the interferometric method as commonly used is subject to a number of limitations and restrictions which become increasingly evident as one goes to higher temperatures.

In the first place the quantity which is ordinarily measured is not a length but a change in length. This feature makes it difficult to determine accurately permanent changes in length which occur during the course of the determination. This objection can, of course, be removed by actually measuring the length by the use of three wave-lengths of light but the complications introduced by this refinement are so great that the scheme has not been widely used. Since the problem is essentially one of obtaining a reasonable cooling rate in the furnace the difficulty can be overcome to some extent by the use of a furnace which has, in addition to its heating element, a helical coil through which some coolant can be passed.

A more serious objection is the necessity of making a correction for the variation with pressure and temperature of the refractive index of air or whatever gas is used as an atmosphere. The correction for change of pressure is, under most conditions, negligible since the variation of pressure during the course of a determination is usually small. The correction for change of temperature is, however, considerable and must always be taken into account.

With small metallic specimens oxidation often causes a large error. In fact, the increase in length due to oxidation can in some cases exceed the increase due to thermal expansion. This error can be estimated if data on two sets of specimens of different length are available but the correction is not entirely satisfactory.[2]

An uncertainty can also be introduced by the presence of an air film between the specimens and the interferometer plate although this can be minimized by careful cutting of the specimens.

The small size of the specimens used makes it necessary that the sample examined be representative of the material as a whole. For non-metallic substances this condition occasionally limits the application of the method, although it usually happens that in such cases none of the common methods is entirely satisfactory. With metals, this difficulty is rarely encountered.

The objection has also been raised that the method, which requires the special attention of the observer throughout the determination, frequently a matter of 6 or 8 hours, is tedious. This is undoubtedly true. However, one must not lose sight of the fact that the value of any datum is a function of the work and care expended in obtaining it and if precise data are desired the results with the interferometer amply compensate for the effort involved.

Two of the most serious objections mentioned, i.e., oxidation of the specimens and correction for the change of refractive index of the atmosphere can be removed if the determination is made in vacuo. For many years the difficulty involved in maintaining a vacuum at high temperatures prevented such an extension of the method but recent advances in high vacuum and in high-temperature technic have removed the most serious obstacles. An attempt

[2] The author is indebted to Mr. G. E. Merritt for the knowledge of this method of correction which was devised by him.

has been made, therefore, to develop a suitable apparatus for measuring linear thermal expansion in vacuum at high temperatures and to test it on suitable metals.

The apparatus as finally decided upon was designed primarily for use with iron and iron alloys but it was felt that the method should be tested first on metals whose behavior is simpler than that of iron, not only to determine its accuracy but to observe its performance under various conditions. Accordingly measurements were made on platinum, gold, magnesium and zinc. These metals were selected because they cover a wide range of properties and because there are existing data against which the results can be checked. Platinum and gold are inert and can be used to check the method at high temperatures; magnesium and zinc are metals which although they melt at a relatively low temperature, are easily oxidized and have high vapor pressures and so provide an opportunity to test the performance of the apparatus under rather severe conditions.[3]

Shortly after the start of the investigation it was found that G. E. Merritt of the U. S. Bureau of Standards had also considered the matter and had even made a design for a furnace. He generously gave the author the benefit of his experience and many features of the present apparatus are the result of his suggestions.

## Apparatus

A general view of the apparatus is shown in Fig. 1. It consists essentially of a vacuum chamber *C* of vitreous silica inserted in a wire-wound furnace *F*. The temperature is measured by a fused silica refraction thermometer *T* which also serves as the lower plate of the interferometer.

The furnace core is an alundum tube *A*, 4 cm inside diameter and 30 cm long, wound with a helical coil of 17 feet of Nichrome IV ribbon, which is held in place by a layer of alundum cement. This core is mounted vertically in a sheet-brass jacket and is surrounded by powdered Sil-O-Cel. It is supported at the base by a plate of Sil-O-Cel mixed with Portland cement and is held in place at the top by a piece of transite board *B*. The whole structure is mounted on a cast bronze base fitted with leveling screws.

The furnace is capable of producing a temperature of 1100° but the upper working limit of the whole apparatus is fixed at approximately 1000° by the refraction thermometer which tends to undergo irreversible changes at higher temperatures. This maximum is not a serious limitation since the behavior of most metals above 1000°C is regular and can be extrapolated with reasonable safety.

The vacuum chamber, which fits loosely into the furance core is a tube of fused silica closed at its lower end and having a side arm sealed on 4 cm below the upper end. The walls of the tube at the upper end are ground and polished to take the window *W*, also of polished fused silica.

[3] Pure iron has also been examined and the apparatus has proved satisfactory in every respect. The results are not included here but will be discussed in a separate paper since they raise a number of questions deserving special comment.

In order to keep a vacuum at high temperatures it is necessary to surround the upper part of the tube with a cooling coil of copper tubing. In the original construction thermal contact between the coil and the tube was secured by pouring a low-melting bismuth-tin alloy over the coil. More satisfactory results can be obtained, however, by the use of a zinc cement. This cement[4] which makes an excellent conducting medium for use up to 150°C is made as follows. Mix two parts of granulated zinc (20 mesh) with one part of powdered zinc. Make the mixture into a paste by adding a binder of one part of technical sodium silicate diluted with one part of water. This is rammed into place and allowed to set, which takes several hours. It is then dried carefully with a hand torch.

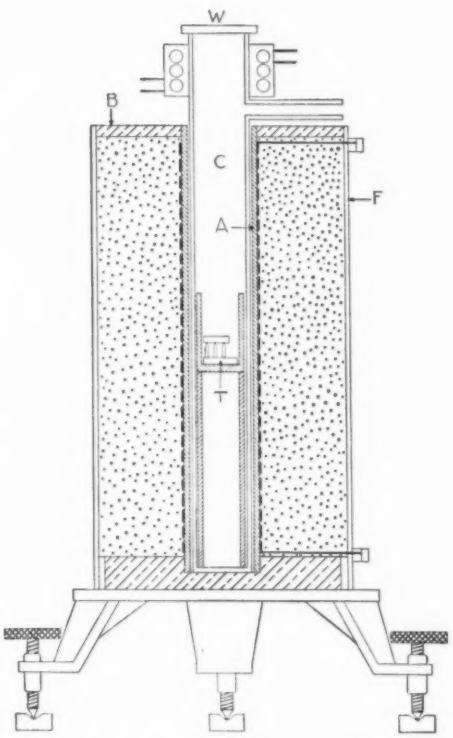



Fig. 1. Furnace for expansion measurements in vacuum.

The side arm of the vacuum chamber is connected through a joint sealed with vacuum-melted wax to a Pyrex vacuum train which consists essentially of a Langmuir mercury condensation pump backed by an oil pump. The vacuum is measured with a McLeod gauge which can be read to $0.5 \times 10^{-3}$ mm.

The window *W* is usually joined to the top of the vacuum chamber with Lubriseal grease. If the window and tube are well ground and polished and if care is exercised in applying a thin layer of grease smoothly and uniformly, the joint will remain tight even at high temperatures.

[4] I am indebted to Mr. H. C. Vacher of the U. S. Bureau of Standards for the formula of this cement.

With such an arrangement it has been possible to obtain a vacuum of less than 0.5$\mu$ at room temperature and to hold it up to 600°C. Above this temperature the grease softens in spite of the cooling coil so that while a vacuum of 0.5$\mu$ has on several occasions been maintained up to 900° a pressure of 1.0 to 2.0$\mu$ is more common. A new joint should be made after each heating however, as there is a tendency for the grease to separate on cooling after heating.

The combination of two plates (which will hereafter be called the interferometer as opposed to the lower plate alone which will be called the thermometer) is contained in a small vitreous silica bucket provided with an optically flat bottom. The bucket, which is raised or lowered by means of a platinum wire, is held in the middle of the furnace by a smaller tube of fused silica on which it rests.

The temperature is determined with a refraction thermometer of the sort described by Luckiesch, Holladay and Sinden.[5] It consists of a fused silica plate which also forms the lower plate of the interferometer. It is a disk 27 mm in diameter and 4 mm thick, the top and bottom surfaces of which are ground and polished at such an angle that approximately six fringes are visible when the plate is illuminated. Two-thirds of the bottom surface is then roughened leaving the fringes visible only on the remaining third. As the index of refraction of the silica changes with temperature these fringes move and their passage across a reference mark scratched on the upper surface of the plate serves to indicate the temperature. With a well-annealed plate of thickness sufficient to hold a flat surface (3 mm is about the minimum) the movement of these fringes is reproducible to a high degree. The calibration of the fringes will be described in detail in a later section. The chief advantage of such a thermometer is that it is in intimate contact with the specimens and should register their temperature quite accurately.

In addition to providing a measure of the actual temperature in the furnace the fringes also indicate to some degree the uniformity of temperature across the diameter. Thus if the temperature distribution across the furnace is non-uniform the index of refraction of the silica will not be the same throughout the plate and the fringes instead of being straight will show a curvature. The gradient along the axis of the furnace has not been determined exactly, but is believed to be small at the center.

The specimens under examination, usually in the form of rods pointed at one end and with miniature feet cut out of the base to reduce the error arising from air films, stand on the part of the thermometer not showing fringes. They support the upper plate of the interferometer which is fused silica disk 15 mm in diameter and 3 mm thick.

Light for the interferometer is provided by a helium discharge tube operated in a standard Keuffel and Esser viewing apparatus. This apparatus includes a filter which screens out all of the light except the wave-length 5877.2A (in vacuo). A view of the interferometer as seen through the viewing apparatus is shown in Fig. 2.

[5] Luckiesch, Holladay and Sinden, J. Franklin Inst. **194,** 251 (1922).

The usual procedure in making the observations is simple. The interferometer is placed in the vacuum chamber, which is then exhausted. The furnace is heated at the rate of 3 to 3.5 degrees per minute and observations of the two sets of fringes are taken; usually the thermometer is read on the passage of every fifth fringe of the interferometer.

## Calibration of the Thermometers

Three refraction thermometers were used during the course of the investigation. Since their history is of some importance in the matter of calibration a brief description of each is given.

The first, No. 12, a disk 27 mm in diameter and 4.438 mm thick, was calibrated by G. E. Merritt by the following procedure. The interferometer, with specimens of a metal of standard melting point[6] was placed in a furnace such as is commonly used for expansion work.[7]

Fig. 2. Typical appearance of the two sets of fringes as seen through the viewing apparatus.

On heating the furnace the temperature corresponding to each fringe of the thermometer was measured with a platinum-platinrhodium thermocouple whose junction was located just beneath the thermometer. The thermocouple and thermometer were then checked on the melting point of the standard metal. The fringe corresponding to the melting point is easily ascertained since the fringes of the interferometer disappear at that point.

A series of such runs was made with the melting points of Sn, Zn, Al and Ag, and the $\alpha-\beta$ inversion of quartz as fixed points. The readings of the thermocouple were then corrected where necessary, plotted against the fringes and a preliminary curve drawn through them. A plot of $dF/dT$, the number of fringes per degree, was then made from the observations and a smooth curve drawn. The values of the slope obtained from this curve were then used to correct and smooth the original curve. This final curve was taken as the calibration of the thermometer.

After this thermometer had been used approximately 20 times, including at least four runs above 900° it was again checked on the melting point of Ag and the $\alpha-\beta$ inversion of quartz. No change or departure from the calibration was detectable.

[6] In order to minimize the damage to the surface of the thermometer by the standard metal when molten, only one of the three specimens was made of it; the other two specimens were of some high-melting metal such as platinum.

[7] Merritt, Sci. Papers Bur. Standards **19**, No. 485, 357 (1924) or cf. Austin, J. Amer. Cer. Soc. **14**, 795 (1931).

The second thermometer No. 16, 27 mm in diameter and 4.438 mm high, was also calibrated by G. E. Merritt by the same procedure except that it was not checked on the melting point of Ag. Its calibration curve was identical within the error of observation with that of No. 12.

A third thermometer, No. 311, 27 mm in diameter and 4.650 mm in thickness, was calibrated against a platinum-platinrhodium couple in much the same manner except that only one fixed point, the $\alpha-\beta$ inversion of quartz was used. The thermocouple was only 0.3°C low at this point which corresponds to less than 0.1 fringe so no correction was applied to the couple readings.

The method of calibration based on melting points has several disadvantages, perhaps the most obvious one being the consumption of a considerable length of time. This is not so serious, however, as the fact that the molten metals are apt to attack the polished surfaces of the interferometer plates so that the possibility of damage to the thermometer is always present, particularly at the higher temperatures. An alternative method based on the change of refractive index of vitreous silica has therefore been devised.

This method is based on the following simple theory. Let us consider a thermometer of thickness $L$ at an initial temperature $t$. It is illuminated with monochromatic light of wave-length $\lambda_v$ in vacuum and $\lambda_1$ in vitreous silica at $t_1$. If the temperature is raised to $t_2$ the thickness of the thermometer becomes $L+\Delta L$ and the light will have a new wave-length $\lambda_2$ in the silica. In this change a number of fringes will pass the reference mark. The precise number of these fringes depends on the change in length $\Delta L$ and on the change of wave-length of the light or its equivalent the change in refractive index of vitreous silica. To be exact, one fringe will pass the mark every time the total number of wave-lengths in the path, i.e., in the double thickness of the thermometer, increases by one and this is true whether the change be due to expansion or to a change of refractive index.

The number of fringes passing due to expansion alone is given by

$$\Delta N = 2(L + \Delta L)/\lambda_v - 2L/\lambda_v. \tag{1}$$

The number of passages due to both factors can be obtained from (1) by correcting it for the change in wave-length.

Thus

$$\Delta N = 2(L + \Delta L)/\lambda_2 - 2L/\lambda_1 \tag{2}$$

$$= 2[((L + \Delta L)/\lambda_v)(\lambda_v/\lambda_2) - (L/\lambda_v)\cdot(\lambda_v/\lambda_1)] \tag{3}$$

$$= 2[((L + \Delta L)/\lambda_v)n_2 - (L/\lambda_v)n_1] \tag{4}$$

where $n_1$ and $n_2$ are the indices of refraction of vitreous silica at temperatures $t_1$ and $t_2$ respectively.

Eq. (4) can be further simplified by replacing $n_2$ by $(n_1+\Delta n)$ which leads to

$$\Delta N = 2[L\Delta n/\lambda_v + \Delta L n_1/\lambda_v + \Delta L\Delta n/\lambda_v]. \tag{5}$$

The last term in the bracket is very small compared to the other two and can

be neglected without appreciable loss of accuracy. Thus by dropping it and expressing $\Delta L$ in terms of the expansion coefficient $\alpha$ we obtain

$$\Delta N = (2L/\lambda_v)[\Delta n + \alpha t n_1]. \tag{6}$$

If $n_1$ and $\alpha$ are known and $\Delta n$ can be expressed as a function of $t$ the thermometer can be calibrated directly by a simple calculation.

The thermal expansion of vitreous silica has been studied by a number of investigators whose results are reviewed by Sosman.[8] Though the data show among themselves variations as large as ten percent they are nevertheless satisfactory for the present purpose since the total espansion is in itself small. In fact it is almost negligible, so that even a relatively large error is unimportant. The value $0.50\times10^{-6}$ has been selected as a fair one for the coefficient of expansion and it has been assumed constant from 20° to 1000°C. This assumption is, strictly speaking, not justified but the error which it introduces is of the same order of magnitude as the possible error in the datum chosen for the coefficient and calculation shows that, if the largest possible error arising from both is assumed, an uncertainty of approximately one-fourth of a fringe is introduced. This uncertainty corresponds, as will be shown later, to 1° or less, which is usually less than the accuracy of the thermometer.

Unfortunately the data on the index of refraction of vitreous silica at high temperatures are rather meager. In fact, the measurements of Rinne,[9] who determined the relation of the refractive index of a vitreous silica prism (source not stated) to temperature from $-160°$ to 1000°C for 4 visible frequencies of helium, including the line at $\lambda5875$, appear to be the only ones available. His results, recalculated to absolute indices, i.e., the index relative to a vacuum, are given by Sosman.[10] Unfortunately they are good only to the fourth decimal place so that the variation of the index with temperature can be obtained to but one significant figure, a precision not sufficient for the present purpose.

It is possible, however, to calculate the total change in refractive index in passing from $t_1$ to $t_2$ from the calibration curves of the three thermometers by the use of Eq. (6) in the form

$$\Delta n = (\lambda_v/2L)(\Delta N + \alpha t n_1). \tag{7}$$

It is necessary to have a value of $n_1$ for the helium line $\lambda_v = 5877.2$A and for this purpose the figure 1.4592, Rinne's value for vitreous silica at 18°C corrected to vacuum, is sufficiently accurate since the second term in the parenthesis is but a small correction to account for the expansion of the thermometer.

The calculation of $\Delta n$ has been performed for each thermometer and the results are given in Table I. The agreement is on the whole satisfactory, the maximum deviation from the mean of the three determinations being $5\times10^{-5}$

[8] Sosman, *Properties of Silica*, Chemical Catalog Co., New York, 1927. Chap. XX.

[9] Rinne, Neues Jahrb. Mineral. Beil. **39**, 388 (1914).

[10] Sosman, reference 8, p. 687; also International Critical Tables VI, 342.

and occurring but three times. The great majority of deviations are equal to or under $\pm 3\times10^{-5}$ which corresponds to an uncertainty in temperature of approximately $\pm 3$°C. While this figure leads to a large error at low temperatures it is quite good enough for most of the temperature range investigated.

TABLE I. *The temperature variation of absolute refractive index of vitreous silica for the helium line* $\lambda_v 5877.2$.

| $t$°C | $\Delta n$ (18° to $t$°) | | | $\Delta n$ | $\Delta n/\Delta t$ | $n_2$ * Absolute index |
|---|---|---|---|---|---|---|
| | No. 12 | No. 16 | No. 311 | (ave.) | | |
| 50 | $25\times10^{-5}$ | $22\times10^{-5}$ | $25\times10^{-5}$ | $24\times10^{-5}$ | | 1.45950 |
| 100 | 74 | 74 | 76 | 75 | $10.0\times10^{-6}$ | 1.46001 |
| 150 | 125 | 130 | 128 | 127 | 10.6 | 1.46053 |
| 200 | 180 | 183 | 186 | 183 | 11.3 | 1.46109 |
| 250 | 238 | 241 | 245 | 241 | 12.0 | 1.46167 |
| 300 | 300 | 309 | 307 | 305 | 12.9 | 1.46231 |
| 350 | 367 | 376 | 373 | 372 | 13.8 | 1.46298 |
| 400 | 438 | 442 | 438 | 439 | 13.9 | 1.46365 |
| 450 | 506 | 510 | 509 | 508 | 13.9 | 1.46434 |
| 500 | 577 | 580 | 579 | 579 | 14.1 | 1.46505 |
| 550 | 647 | 652 | 651 | 650 | 14.4 | 1.46576 |
| 600 | 721 | 724 | 724 | 723 | 14.7 | 1.46649 |
| 650 | 794 | 796 | 795 | 795 | 14.6 | 1.46721 |
| 700 | 867 | | 870 | 868 | 14.8 | 1.46794 |
| 750 | 942 | | 945 | 944 | 15.0 | 1.46870 |
| 800 | 1017 | | 1020 | 1018 | 15.2 | 1.46944 |
| 850 | 1094 | | 1095 | 1094 | 15.4 | 1.47020 |
| 900 | 1171 | | 1176 | 1174 | 15.4 | 1.47100 |
| 950 | 1248 | | | 1248 | | 1.47174 |

* Based on the value 1.45926 at 18°C.

Moreover it can be materially reduced by checking the thermometer at one or more fixed points, a precaution which should be observed in any precise measurement.

Having secured the desired values of $\Delta n$, all the data necessary for the calibration of the thermometer are at hand. Inserting the appropriate values of the constants in (6) we obtain

$$\Delta N = (L(\text{in cm})/2938.6 \times 10^{-8})[\Delta n + 0.7296 \times 10^{-6} t]. \qquad (8)$$

This equation may be used in either of two ways. First $\Delta N$ for any given temperature may be calculated by inserting the proper values of $t$ and of $\Delta n$ as obtained by interpolation of Table I. If a complete calibration table is to be made $\Delta N$ can be reckoned for every 50° and if the values so found are plotted against $t$ on a large scale the temperature corresponding to each fringe can be rapidly read off.

The other method is to solve for values of $\Delta n$ for say, every 10 fringes and then to convert them into temperatures by interpolation in Table I. This again leads to a curve of $\Delta N$ *vs.* $t$ which may be used directly or as the basis of a table. By use of these methods to check back on the calibration tables of the three thermometers it was found that the original calibrations could be reproduced with an accuracy of $\pm 2.5$°C.

The indications are strong that the values of $\Delta n$ are representative of well-fused, well-annealed vitreous silica, that is silica which has been fused at a temperature sufficient to give a product showing little tendency to devitrification and which has been properly annealed to remove strains. A glance at the history of the thermometers given in a preceding paragraph shows that they had been subject to quite a variety of conditions yet they still give with reasonable accuracy the same change in $\Delta n$. It is to be particularly emphasized that No. 12 adhered to its original calibration even after considerable use and there is no reason to suppose that it would not continue to do so for a long time provided it is not carried too often to a temperature above 950° or held above this temperature for any extended time. Vitreous silica begins to show an anomalous behavior in the neighborhood of 1000°C and if carried above that temperature it usually suffers some permanent change. The safe limit for continued operation is, as stated before, approximately 950°C.

There must also be considered the possibility of finding a thermometer made of vitreous silica which is not well annealed so that some check on the calibration is necessary. It is also desirable from the point of view of the thermometer scale to have such a check since it is possible to reduce materially the uncertainty of the calibration by checking a few well-chosen points. For this purpose the transition points of solids are particularly suitable since they offer less chance of damage to the thermometer than do melting points. An extremely useful transition of this sort is the $\alpha-\beta$ inversion in quartz occurring on heating at 573.3°.[11] A small plate of quartz polished to show interference fringes can be placed on the thermometer and gives an excellent indication of the transition by the disturbance of the fringes. Some little caution is necessary in the use of such a plate to keep it from shattering because of the large volume change at the inversion. With a heating rate of 3 to 3.5° per minute no difficulty should be encountered.

A study of the behavior of the thermometers discloses a tendency to lag at the start of a run. This tendency is commonly not large and in every case seems to disappear between 50° and 75°C. It is well to take this into account, however, by giving observations above 100°C more weight than those below this temperature.

In passing it is of interest to calculate the actual index of refraction of silica as a function of temperature and to compare the values derived from the thermometers with Rinne's measurements made on a prism by the method of minimum deviation. To make this computation the value 1.4592 used for $n_1$ in Eq. (7) is taken as the absolute refractive index at 18°C. The results are given in the last column of Table I. In Table II the calculated values are compared with Rinne's directly determined points. Since the datum of Rinne at 18° is used as a reference the calculated index is only given to four decimals. The agreement is on the whole satisfactory considering the difficulty of measurement at high temperatures.

There is one significant difference, however; Rinne's points show a small, though apparently real, irregularity in the range 500 to 600°C having no

[11] Bates and Phelps, Sci. Papers Bur. Standards **22**, (No. 557), 315 (1927).

TABLE II. *Comparison of the absolute index of refraction of vitreous silica as determined from refraction thermometers 12, 16 and 311 and by Rinne.*

| $t$°C | $n_2$ Calculated value | $n_2$ Rinne's value | (Author's value − Rinne's) ×100 |
|---|---|---|---|
| 18 | 1.4592 | 1.4592 | 0 |
| 130 | 1.4603 | 1.4604 | −0.1 |
| 235 | 1.4615 | 1.4616 | −0.1 |
| 365 | 1.4632 | 1.4633 | −0.1 |
| 475 | 1.4647 | 1.4649 | −0.2 |
| 590 | 1.4663 | 1.4663 | 0 |
| 1000 | 1.4725 | 1.4729 | −0.4 |

counterpart in the calculated values which lie on a smooth curve. The cause of this apparent anomaly is not clear nor is information concerning the source and history of Rinne's specimens or the details of his methods available. There are two factors which might have caused such behavior. First the vitreous silica may have contained some residue of the structure of the original crystalline quartz whose inversion lies in this temperature range; second, he reports that two thermocouples were used—one of silver-constantan for the range—160° to 600° and another of platinum-platinrhodium for the range 600° to 1000°. Since the anomaly occurs just below 600° it is not impossible that a change of thermocouples may have introduced some uncertainty.

## ERRORS

The application of interferometry to the measurement of changes in length enables one to determine this quantity with a precision and an accuracy much greater than that obtainable in the observation of the other variables such as temperature. It is of interest, therefore, to investigate the relative magnitudes of the various errors to see which factor limits the accuracy of the result by introducing the greatest uncertainty.

The possible error in the observation of the fringes indicating the expansion of the specimen is approximately 0.1 fringe, or 0.05 $\lambda_v$ which for the helium line used corresponds very nearly to $3\times10^{-6}$ cm. The percentage error depends on the actual magnitude of the total expansion but for most metals it is of the order of 1 percent at 100°; 0.1 percent at 500° and 0.05 percent at 1000°. This is generally speaking quite small.

In correcting the total expansion to expansion per unit length the mean length of the three specimens is involved. This can be measured to $\pm0.001$ mm which for the size of specimen usually used corresponds to less than $\pm0.1$ percent.

In the measurement of temperature the thermometer, like the interferometer, can be read to $\pm0.1$ fringe. On the thermometers used the average change per fringe is approximately 5° so that the precision of the temperature measurement is $\pm0.5$°. The accuracy of the thermometers is frequently not as good as this, however. The results of Table I show that variations from the average index corresponding to as much as $\pm3$° are to be expected. At 100° this corresponds to 3 percent, at 300° it is 1.0 percent; and at 600° it is

0.5 percent. Obviously this is the limiting factor at low temperatures and in most cases at high temperatures. If the calibration curve is better than $\pm 3°$ these percentages are considerably lower but it seems unlikely that it will ever be as low as 0.1 percent which is the accuracy obtainable in the length measurements.

Assuming the maximum error which may include all the errors mentioned it should not be greater than 1 percent at 300°, or 0.7 percent at 600° and the probable error will, of course, be much less than this.

## Results

The results of the tests of the apparatus are presented whenever possible in the form recommended by Schad,[12] i.e., the observed expansion with increased temperature is plotted; a table of average expansion coefficients over several temperature ranges is given; a second table showing the instantaneous expansion coefficient in steps of 100° included, and finally an equation giving either the length or expansion coefficient as a function of temperature is furnished.

All data have been corrected to 0° as the initial temperature. As explained in section III there is a tendency for the refraction thermometer to lag up to 50° or 75° so the correction to zero has been made by deriving an equation to fit the experimental observations above 100°C extrapolating this to 0° and then translating the origin to make zero change in length correspond to 0°C.

Values of the change in length have been calculated for every 100° from the equation of the best line through the experimental points and the coefficients are obtained from these.

### Platinum

This metal is particularly well adapted to test the apparatus in the neighborhood of the maximum working temperature. It is inert, its expansion is regular and relatively well known since reliable data are available from room temperature to 1000°C. Platinum does not, of course, provide a test of the prevention of oxidation but it does make it possible to check the accuracy of the thermometers and of the over-all accuracy of the apparatus throughout the entire range of working temperatures. The actual results are shown graphically in Fig. 3 while numerical values appear in Table III.

Preliminary data were obtained with 0.8 mm thermocouple wire in an interferometric apparatus of the usual type such as that used in calibrating the thermometers. Reproducible values for the expansion were obtained if the temperature did not exceed 750°C but above this temperature the metal became too soft to support the weight of the upper plate of the interferometer and an appreciable flow leading to a permanent deformation took place. The magnitude of this flow was variable yet always considerable, as shown by the dashed curve in Fig. 3 which represents a typical example. Under such conditions reproducibility was neither expected nor achieved.

[12] Report on Thermal Expansion of Materials, Appendix II; Report of Joint Committee on Effect of Temperature on the Properties of Metals, Proc. Am. Soc. Testing Mats. **27,** Part I, 153 (1927).

The final specimens were cut from a 3 mm rod of C.P. platinum (99.995 percent) showing only spectroscopic traces of Fe, Ca and Mg. Three pieces of this rod, 4.050 mm in height, were used and proved to be sufficiently sturdy to give reproducible results even at 900°C. After being cut the specimens were heated to 900° and allowed to cool in the furnace to remove any strains before the measurements were taken. With this treatment the results of two different determinations, which included observations during cooling as well as heating, checked within the limit of the experimental error.

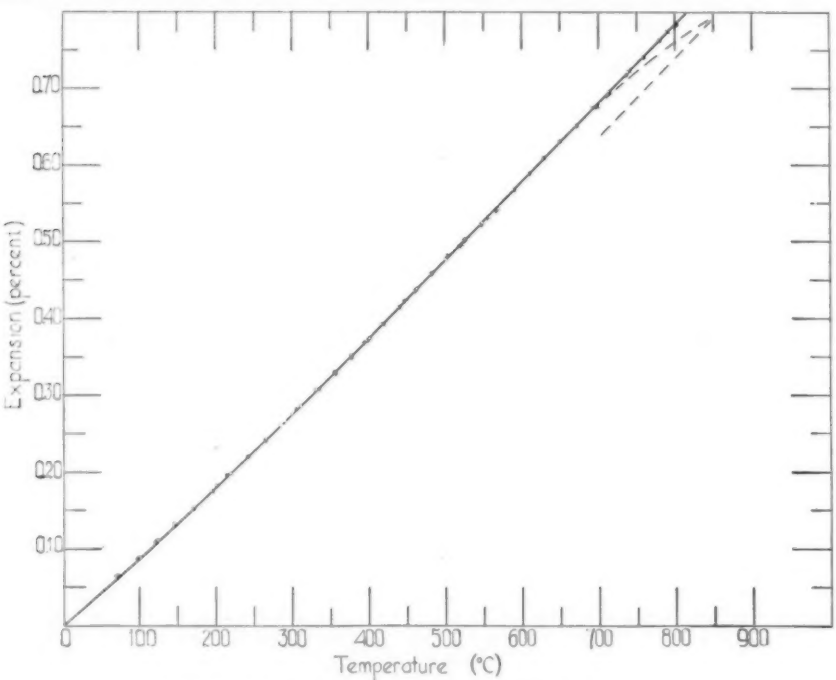



Fig. 3. The linear thermal expansion of platinum.

TABLE III. *Thermal expansion of platinum.*
$l_t = l_0(1 + 8.868 \times 10^{-6}\, t + 1.324 \times 10^{-9}\, t^2)$
$\alpha = dl/dt = (8.868 \times 10^{-6} + 2.648 \times 10^{-9}\, t)$
$\alpha_m = (8.868 \times 10^{-6} + 1.324 \times 10^{-9}\, t)$

| $t$°C | $\Delta l/l_0$ (mm/m) | Mean coef. 0° to $t$° $\alpha_m \times 10^6$ | Instantaneous coef. at $t$° $\alpha \times 10^6$ | $\Delta l/l_0$ Calculated from data of Holborn & Day | Diff. Author's value − H & D's |
|---|---|---|---|---|---|
| 0 | 0 | | | 0 | 0 |
| 100 | 0.899 | 8.99 | 9.13 | 0.900 | −0.001 |
| 200 | 1.825 | 9.12 | 9.39 | 1.827 | −0.002 |
| 300 | 2.775 | 9.25 | 9.66 | 2.780 | −0.005 |
| 400 | 3.760 | 9.40 | 9.93 | 3.759 | +0.001 |
| 500 | 4.770 | 9.54 | 10.19 | 4.765 | +0.005 |
| 600 | 5.800 | 9.66 | 10.46 | 5.797 | +0.003 |
| 700 | 6.845 | 9.78 | 10.72 | 6.855 | −0.010 |
| 800 | 7.855 | 9.82 | 10.97 | 7.941 | −0.09 |
| 900 | 8.945 | 9.94 | 11.27 | 9.045 | −0.10 |

Of the existing measurements on platinum the only reliable ones at high temperatures appear to be those of Holborn and Day[13] which are given for comparison in Table III. The agreement is excellent, the differences in $\Delta l$ being 0.2 percent or less except at 800° and above where it amounts to slightly over one percent. It is interesting to note that it is in just this region that the greatest softening occurs as indicated by the preliminary measurements, and the difference is probably to be explained on this basis. In view of the closeness of this agreement the equation for $\Delta l$ given by Holborn and Day has been retained and is given at the head of Table III.

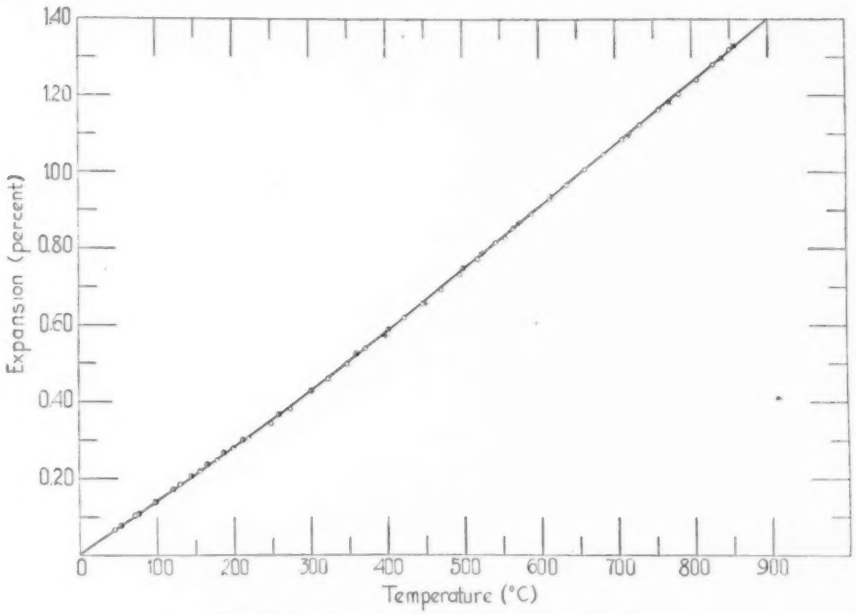



Fig. 4. The linear thermal expansion of gold.

The coefficient of thermal expansion at 20° is therefore the same as that of Holborn and Day, i.e., $8.921 \times 10^{-6}$ which is in good agreement with the work of other investigators at this temperature.

**Gold**

This metal possesses very nearly the same properties as platinum and was selected because it offered the same advantages as a test material. The three specimens, which were cut from a 3 mm rod of fine gold (99.98 percent)were 4.575 mm in height. After being cut they were given the same annealing treatment as the platinum, i.e., they were heated to 900°C and were allowed to cool with the furnace.

Two different determinations, which included measurements during heating as well as cooling gave results which checked within the limit of error of the observations. These results are shown graphically in Fig. 4; the actual data are found in Table IV.

[13] Holborn and Day, Ann. d. Physik **4**, 104 (1901).

TABLE IV. *Thermal expansion of gold.*
$l_t = l_0(1+14.06\times10^{-6}\,t+1.672\times10^{-9}\,t^2+1.197\times10^{-12}\,t^3)$
$\alpha = dl/dt = (14.06\times10^{-6}+3.344\,t+3.591\,t^2)$
$\alpha_m = (14.06\times10^{-6}+1.672\times10^{-9}\,t+1.197\times10^{-12}\,t^2)$

| $t$°C | $\Delta l/l_0$ (mm/m) | Mean coef. 0° to $t$°C $\alpha_m\times10^6$ | Instantaneous coef. at $t$° $\alpha\times10^6$ | $\Delta l/l_0$ As given by Müller | Diff. Author's value −Müller's |
|---|---|---|---|---|---|
| 0 | 0 | | 14.06 | | |
| 100 | 1.424 | 14.24 | 14.39 | 1.44 | −0.02 |
| 200 | 2.888 | 14.44 | 14.82 | 2.92 | −0.04 |
| 300 | 4.401 | 14.67 | 15.28 | 4.44 | −0.04 |
| 400 | 5.968 | 14.92 | 15.78 | 6.01 | −0.05 |
| 500 | 7.598 | 15.20 | 16.34 | 7.62 | −0.02 |
| 600 | 9.296 | 15.50 | 16.94 | | |
| 700 | 11.072 | 15.81 | 17.56 | | |
| 800 | 12.931 | 16.18 | 18.30 | | |
| 900 | 14.881 | 16.54 | 19.02 | | |

The only previous data on the expansion of gold above 100°C are those of Müller for the range 0° to 500°C[14] which are included in Table IV. The agreement is on the whole satisfactory, the maximum difference, which occurs at 200°C, being slightly over one percent. The mean coefficient between 10° and 100°C is $14.24\times10^{-6}$. This is in fairly good agreement with the value of $14.31\times10^{-6}$ obtained by Grüneisen[15] for the range 17° to 100°C.

**Magnesium**

Magnesium was selected for study because it possesses a number of properties which can be used to advantage in testing the apparatus and the method. For example, it can easily be obtained in a state of high purity; it oxidizes readily; it has an appreciable vapor pressure; and finally there appear to be reliable data on its expansion already reported against which the results can be checked.

The specimens actually used were cut from a $\frac{1}{4}$ inch extruded rod of magnesium (99.99 percent). A portion of this rod was turned down to $\frac{1}{8}$ inch diameter and then cut into lengths of approximately 4 mm from which the test pieces were fashioned.

Although the pieces were not annealed before use the first run on each

TABLE V. *Linear thermal expansion of "unannealed" magnesium.*

| $t$°C | $\Delta l/l_0$ (mm/m) | Mean coef. 0° to $t$°C $\alpha_m\times10^6$ | Instantaneous coef. at $t$°C $\alpha\times10^6$ |
|---|---|---|---|
| 0 | 0 | | |
| 50 | 1.20 | 24.0 | 24.5 |
| 100 | 2.45 | 24.5 | 25.4 |
| 150 | 3.74 | 25.0 | 26.5 |
| 200 | 5.10 | 25.5 | 27.8 |
| 250 | 6.52 | 26.1 | 29.1 |
| 300 | 8.01 | 26.7 | 30.8 |
| 350 | 9.60 | 27.4 | 32.4 |
| 400 | 11.25 | 28.1 | 34.5 |
| 450 | 13.05 | 29.0 | 36.5 |
| 500 | 14.90 | 29.8 | 38.6 |

[14] Müller, Phys. Zeits. **17,** 29 (1916).
[15] Grüneisen, Ann. d. Physik **33,** 65 (1910).

set of specimens was carried to at least 250°C which was considered sufficiently high to remove the strains introduced during the extrusion of the rod or during its subsequent machining. The results obtained on these first runs are reported in Table V under the heading of "unannealed" metal. They are also shown by the dashed line in Fig. 5. Determinations made after the first heating have been considered as applying to "annealed" metal and the data under that caption in Table VI and shown in Fig. 5 are those for metal which had been heated at least once to above 250°C.

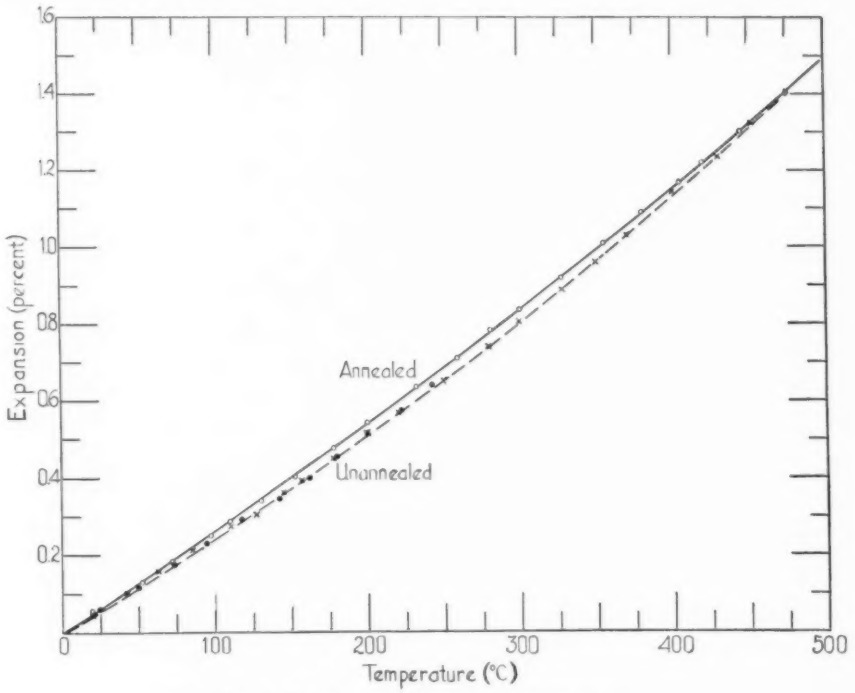



Fig. 5. The linear expansion of magnesium.

TABLE VI. *Linear thermal expansion of "annealed" magnesium.*

$l_t = l_0[1+(24.80\ t+0.00961\ t^2)\times 10^{-6}]$

$\alpha = dl/dt = (24.80+0.01922\ t)\times 10^{-6}$

$\alpha_m = (24.80+0.00961\ t)\times 10^{-6}$

| $t$°C | $\Delta l/l_0$ in mm/m | Mean coef. 0 to $t$°C $\alpha_m\times 10^6$ | Instantaneous coef. at $t$°C $\alpha\times 10^6$ | $\Delta l/l_0$ Calculated from equation of Hidnert & Sweeney | Mean coef. 20° to $t$°C $\alpha_m\times 10^6$ Hidnert & Sweeney |
|---|---|---|---|---|---|
| 0 | 0 | — | 25.00 | 0 | |
| 50 | 1.30 | 25.50 | 26.0 | 1.29 | |
| 100 | 2.60 | 26.0 | 26.9 | 2.59 | 26.0 |
| 150 | 3.95 | 26.4 | 27.8 | 3.97 | |
| 200 | 5.35 | 26.9 | 28.8 | 5.39 | 26.9 |
| 250 | 6.85 | 27.3 | 29.8 | 6.84 | |
| 300 | 8.37 | 27.9 | 30.7 | 8.36 | 27.9 |
| 350 | 9.95 | 28.3 | 31.6 | 9.95 | |
| 400 | 11.53 | 28.8 | 32.6 | 11.53 | 28.8 |
| 450 | 13.15 | 29.3 | 33.5 | 13.16 | |
| 500 | 14.90 | 29.8 | 34.5 | 14.88 | 29.8 |

It is apparent from the figure that the annealed specimens have a slightly greater expansion than do the unannealed ones. There is another difference, however, which is not evident from the figure. The relatively small size of the figure and the fact that the curves as shown have been smoothed a little have tended to conceal the fact that the expansion of the unannealed metal is not regular above 350°C but is marked by a series of three or four little steps while on the other hand the annealed metal gives results which may be represented by a smooth curve with much greater accuracy. These conditions indicate that the steps are caused by the release of strains in the metal and therefore show that 350°C is probably well above the annealing temperature.

Above 500°C the unannealed specimen showed a more rapid increase in length than did those which were annealed. Such a rise in the expansion coefficient, which is certainly not what one would predict, cannot be laid to oxidation since the surface of the metal was untarnished at the end of the run but may have arisen through some error introduced by the condensation of magnesium in the window as will be described later.

No attempt was made to use single crystals of magnesium since the work of Schmid[16] and of Bridgman[17] has shown that although it crystallizes in the hexagonal system it is practically isotropic insofar as expansion is concerned. For example, in the temperature range 20° to 100°C the coefficients parallel and perpendicular to the principal axis of the crystal are $\alpha_{\parallel}=26.4\times10^{-6}$ and $\alpha_{\perp}=25.6\times10^{-6}$ according to Schmid while Bridgman gives $\alpha_{\parallel}=27\times10^{-6}$ and $\alpha_{\perp}=24.3\times10^{-6}$.

The existing data on polycrystalline magnesium have been well reviewed by Hidnert and Sweeney[18] who have themselves contributed an excellent set of measurements for temperatures up to 500°C. Their values for the average coefficient for a series of temperature ranges is included in Table VI. The agreement is excellent although it must be remembered that their coefficients are referred to 20° whereas the author has used 0°.

Some little difficulty was experienced in preliminary tests because of the softness of the metal at elevated temperatures. The use of specimens in the form of pointed rods 3 mm in diameter and 4 to 5 mm high in the final measurements gave satisfactory results, however. The chief difficulty encountered with magnesium was the vaporization of the metal at the higher temperatures. Above 500°C the deposits of metal on the window and on the cool portions of the vacuum chamber became so thick that it was difficult to carry on the measurements. When an attempt was made to determine the contraction of the metal on cooling the condensation became so great that further study was impossible. The observations as reported are, therefore, derived entirely from data obtained on heating.

This vaporization was, on the whole, much greater than one would expect from the behavior of magnesium under ordinary conditions, that is, in air at one atmosphere. The explanation appears to be relatively simple; the

[16] Schmid, Z. Elektrochemie **37,** 447 (1931).
[17] Bridgman, Proc. Amer. Acad. Arts Sci. **67,** 29 (1932).
[18] Hidnert and Sweeney, Bur. Standards J. Res. **1,** 771 (1928).

mean free path of the molecules of the magnesium vapor in vacuo is much greater than it is in air at one atmosphere hence the atoms or molecules of the vapor have a better opportunity to migrate to the window and cold portions of the vacuum chamber where they condense. The film thus formed gradually builds up until it becomes opaque and makes further measurement impossible. Fortunately these films can easily be removed with nitric acid, but the necessity of cleaning out the chamber between each run is always annoying.

A careful examination of several of these films disclosed the presence of a little oxide. The specimens, however, were as bright at the end of each run as they were at the beginning. Although free from oxidation the specimens did show clearly that the vaporization had not been uniform over the surface of the metal but had been distinctly selective. The whole surface appeared to be strongly etched and in many cases deep pits had formed. In one instance a large hole had been eaten almost through the specimen so that it started to collapse at a temperature of 550°C thus preventing further measurement.

**Zinc**

In spite of the fact that zinc melts at the relatively low temperature of 419°C so that it does not test the performance of the apparatus at elevated temperatures it offers what are in many respects the most severe conditions provided by any of the metals studied. It oxidizes readily, has a high vapor pressure below the melting point and, unlike magnesium, possesses an expansion which is highly anisotropic. This last property makes it difficult to obtain values for the expansion of different samples of polycrystalline metal since any tendency to preferred orientation of the crystal grains in a given specimen influences the expansion to a marked degree. The effect produced by such orientation is frequently much greater than that produced by the presence of impurities and the rather large differences found among the values reported in the literature for the expansion of polycrystalline metal are probably to be explained by differences in microstructure.

Because of this anisotropic behavior and because it was desired to obtain further information on the alleged existence of inversion points in zinc it was considered desirable to study the expansion of single crystals. Accordingly measurements were made on five such crystals in the form of rods approximately 2.5 mm in diameter.

Four of these crystals were obtained through the courtesy of Professor P. W. Bridgman and were selected from a number prepared by him for studies of the electrical properties of zinc. Each crystal was oriented in a different direction, that is in no two of them did the principal axis of the crystal make the same angle with the axis of the rod.

The zinc from which they were made was "Kahlbaum's best." Since the purity of such material is a little indefinite one of the crystals ($\theta = 52°$) was analyzed after use by the spectrographic method through the courtesy of the New Jersey Zinc Company. The results indicated the presence of 0.014 percent of lead and 0.012 percent of cadmium but aside from these two the zinc appeared to be of high purity. There is a possibility that a trace of iron was also present since this was not determined in the analysis.

The method of preparation of the crystals and a number of properties for similar ones made from the same metal are fully described by Bridgman.[19]

The fifth crystal was made by the author from a sample of "spectroscopically pure" zinc (99.999 percent) obtained through the courtesy of the New Jersey Zinc Company. The method of preparation was the same as that used by Bridgman except that the mold had but one branch.

The specimens for use in the interferometer were cut from the crystals by cleaving off lengths of the rod. This is ordinarily a somewhat hazardous procedure for although the crystals show a good cleavage along the basal plane they are so soft that there is always danger of deforming the rod and of destroying the single-crystal structure in the act of cutting. If, however, the crystal is first dipped in liquid air, it cleaves easily and with no damage. Three specimens approximately 4 mm in length were cut from each crystal.

The expansion was measured across the diameter of the rod, that is in the direction *CD* in Fig. 6. By making the measurements in this direction it was

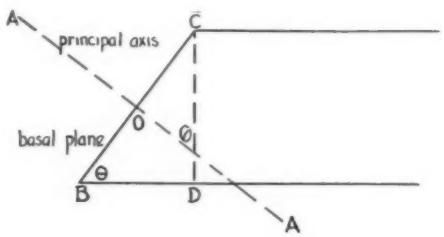



Fig. 6. Diagram illustrating the method of orienting a crystal.

possible to obtain specimens of approximately the same height without working the crystal. It was still necessary, however, to smooth off the points on the crystal which were in contact with the fused silica plate but this could be easily accomplished without destroying the specimen.

In one case a set of single crystal specimens with the direction CD parallel to the principal axis was very badly deformed in the cutting. Measurements made on them gave results which indicated that the rod was no longer a single crystal hence observations made on it are listed under the caption "polycrystalline metal."

The orientation in the crystal of the direction in which the expansion was measured was obtained by measuring the angle between the cleavage plane and the axis of the rod, that is, the angle *DBO* in Fig. 6. This was done by clamping the crystal on the revolving stage of a petrographic microscope in such a way that the basal plane was vertical. One of the cross hairs of the microscope was then set on the image of the side of the rod and the angle between it and the cleavage plane determined by rotating the stage until the cross hair coincided with the edge of the basal plane. A number of such readings were made on each crystal and the average taken as the value of the angle.

[19] Bridgman, Proc. Amer. Acad. Arts Sci. **63,** 351 (1929).

From the geometry of the crystal (Fig. 6) it is evident that the angle between the direction in which the expansion was measured and the principal axis, that is the angle *ODC*, is equal to angle *DBO*, therefore, the angle as read from the stage of the microscope gives the desired value directly.

The results of the measurements are given in Fig. 7 and Tables VII, VIII and IX. They represent observations made on the first heating and do not include any measurements on cooling for reasons which will be discussed in the next paragraph.

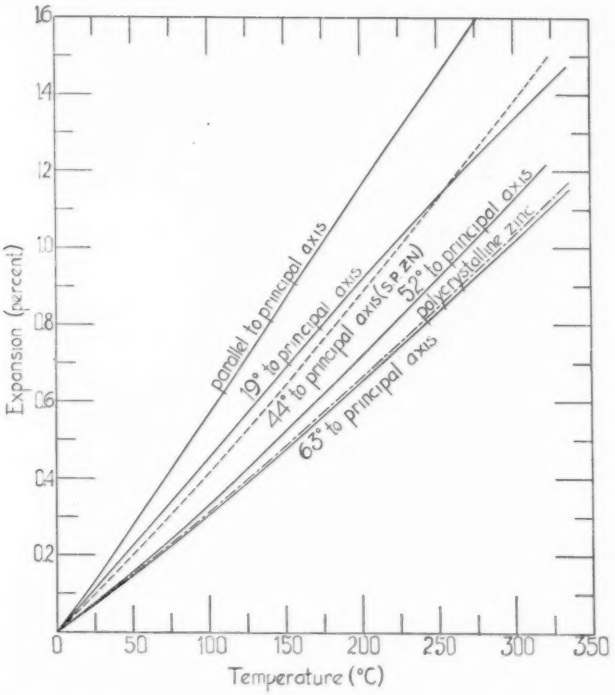



Fig. 7. The linear thermal expansion of single-crystal zinc for various orientations in the crystal. Unbroken curves, Kahlbaum Zn. Broken curve, spectroscopically pure Zn. Curve with long and short dashes, polycrystalline Zn.

TABLE VII. *Length changes in single-crystal and polycrystalline zinc.*

| $t$°C | $\Delta l/l_0$ (mm/m) | | | | | |
|---|---|---|---|---|---|---|
| | ‖ prin. axis | 19° to prin. axis | 44° to prin. axis* | 52° to prin. axis | 63° to prin. axis | Poly-crystalline |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 2.73 | 2.21 | 2.00 | 1.59 | 1.50 | 1.55 |
| 100 | 5.70 | 4.58 | 4.04 | 3.31 | 3.00 | 3.12 |
| 150 | 8.60 | 6.97 | 6.37 | 5.21 | 4.72 | 4.85 |
| 200 | 11.61 | 9.18 | 8.80 | 7.18 | 6.49 | 6.61 |
| 250 | 14.52 | 11.38 | 11.38 | 9.19 | 8.30 | 8.44 |
| 300 | 17.20 | 13.33 | 13.82 | 11.25 | 10.18 | 10.33 |
| 350 | — | 15.32 | — | 13.34 | 12.06 | — |

* Spectroscopically pure zinc.

TABLE VIII. *Mean linear expansion coefficients of single-crystal zinc.*

| $t$°C | $\alpha_m \times 10^6$ from 0 to $t$°C | | | | | |
|---|---|---|---|---|---|---|
| | ∥ prin. axis | 19° to prin. axis | 44° to prin. axis | 52° to prin. axis | 63° to prin. axis | Poly-crystalline |
| 50 | 54.6 | 44.2 | 40.0* | 32.0 | 30.0 | 31.1 |
| 100 | 57.0 | 45.8 | 40.4 | 33.1 | 30.0 | 31.2 |
| 150 | 57.4 | 47.0 | 42.3 | 34.8 | 31.5 | 32.3 |
| 200 | 58.0 | 45.9 | 44.0 | 35.9 | 32.5 | 33.0 |
| 250 | 58.1 | 45.5 | 45.5 | 36.7 | 33.2 | 33.8 |
| 300 | 57.3 | 44.4 | 46.0 | 37.5 | 33.9 | 34.4 |
| 350 | — | 43.7 | — | 38.0 | 34.5 | — |

* Spectroscopically pure zinc.

TABLE IX. *Instantaneous expansion coefficients of single-crystal zinc.*
Values in parenthesis are obtained by extrapolation.

| $t$°C | $\alpha \times 10^6$ at $t$°C | | | | | | |
|---|---|---|---|---|---|---|---|
| | ∥ prin. axis | 19° to prin. axis | 44° to prin. axis | 52° to prin. axis | 63° to prin. axis | ⊥ prin. axis | Poly-crystalline |
| 50 | 57.0 | 45.8 | 40.4* | 33.1 | 30.0 | (16.0) | 31.2 |
| 100 | 58.7 | 47.6 | 43.7 | 36.2 | 32.2 | (19.0) | 33.0 |
| 150 | 59.1 | 46.0 | 47.6 | 38.7 | 34.9 | (21.8) | 34.9 |
| 200 | 59.2 | 44.1 | 50.1 | 39.8 | 35.8 | (23.8) | 35.9 |
| 250 | 55.9 | 41.5 | 50.2 | 40.7 | 36.9 | (25.5) | 37.2 |
| 300 | | 39.4 | | 41.5 | 37.6 | (27.0) | |

* Spectroscopically pure zinc.

In making the measurements it was found that, just as in the case of magnesium, the chief difficulty arose from the vaporization of the metal. Above 300°C this became quite serious, frequently resulting in the appearance of an opaque film of zinc on the window. Whenever an attempt was made to study the behavior of the metal on cooling, the deposit on the window and on other cold portions of the vacuum chamber became very thick, forming an almost perfect mirror which made further measurement impossible; it also made it necessary to clean out the apparatus after each determination. An examination of these films disclosed the presence of a trace of zinc oxide but the greater part of the deposit was metallic zinc.

The specimens themselves showed little or no evidence of oxidation, the cleavage planes being almost without exception as bright at the end of a run as they were at the beginning. They did show, however, a certain amount of selective vaporization which produced a sort of etching figure on a few of the cleavage faces. This effect was particularly marked on the polycrystalline specimens which showed appreciable pitting. From these results one is led to wonder whether the vaporization would be the same from different faces of a perfect strain-free crystal; or indeed, whether such a crystal has a vapor pressure.

The only existing data on single-crystal zinc with which the present values can be compared[20] are those of Bridgman[21] and Grüneisen and Goens.[22]

[20] The x-ray determinations of the expansion of single crystals made by McLennan and Monkman, Trans. Roy. Soc. Can. Sect. III (3) **23,** 255 (1930) give only the mean coefficient between −190° and +18°C.

The former, who investigated crystals made from zinc which must have been very similar to that used in making the crystals employed in the present study, obtained the values $\alpha_{\parallel} = 57 \times 10^{-6}$ and $\alpha_{\perp} = 12.6 \times 10^{-6}$ at room temperature. The latter found $63.9 \times 10^{-6}$ and $14.1 \times 10^{-6}$ respectively for $\alpha_{\parallel}$ and $\alpha_{\perp}$ for the temperature interval 20° to 100°C.[23]

Only one of these coefficients, $\alpha_{\parallel}$ was determined experimentally in the present investigation. The value observed, $57 \times 10^{-6}$, for the temperature interval 0° to 100°C, is in excellent agreement with Bridgman's datum. It is true that the temperature interval is different in the two cases but it has long been known that this coefficient shows but little variation with temperature in this region so that the objection is not serious. The observation of Grüneisen and Goens comes approximately 10 percent higher; moreover, their data indicate that the coefficient is actually decreasing while the author's results show a slight increase, at least up to 150°C (cf. Fig. 9). A comparison of the two investigations reveals no satisfactory explanation for this lack of agreement.

The quantity $\alpha_{\perp}$ which was not determined directly by the author, can be obtained indirectly by extrapolation of the other data. This can be most easily accomplished by the use of the cosine-square law which states that the expansion coefficient in a crystal belonging to the hexagonal system is a linear function of the square of the cosine of the angle $\theta$, that is, the angle between the direction of measurement and the principal axis of the crystal. Such a plot, of $\alpha_m$, the mean coefficient from 0° to 100°C *vs.* $\cos^2 \theta$ is given in Fig. 8. From this graph $\alpha_{\perp}$ is found to be approximately $19 \times 10^{-6}$.

At first glance this appears to be in very poor agreement with the values of previous investigators. The discrepancy is not so great as it appears, however, since it is well known that $\alpha_{\perp}$ varies considerably with temperature, and therefore one can justifiably compare only coefficients for the same temperature. A similar plot for $\alpha_{\perp}$ between 0° and 40°, which is the same as the true coefficient at 20° and therefore is comparable with Bridgman's data at room temperature, yields the value of approximately $15 \times 10^{-6}$. This is still over 10 percent higher than Bridgman's value. A similar plot for comparison with Grüneisen and Goens yields a figure of nearly $18 \times 10^{-6}$ for the temperature interval in which their datum holds. It must be remembered, however, that in all these extrapolations the line extended is based on but five points and as Fig. 8 clearly shows there is a considerable amount of arbitrary selection in choosing the course of the line. Since a small variation in the slope of the line produces a large variation in its intercept the error in the extrapolated figures can be quite large and might easily account for the difference between value of Bridgman and that of the author. It does not seem quite large enough to explain the poor agreement with the data of Grüneisen and Goens.

[21] Bridgman, Proc. Nat. Acad. Sci. **10,** 411 (1924).

[22] Grüneisen and Goens, Zeits. f. Physik **29,** 141 (1924).

[23] It should be noted in this connection that the International Critical Tables **II,** p. 462 ascribe to Grüneisen and Goens a value of $15.6 \times 10^{-6}$ for $\alpha_{\perp}$ at 60°C. This appears to be the figure actually found for the direction $\theta = 81°$ and does not therefore represent the true $\alpha_{\perp}$ which is the lower value already given.

Besides its usefulness in extrapolation the method of plotting used in Fig. 8 gives a fair indication of the consistency of the data. Thus it appears probable that the coefficient for the direction $\theta = 19°$ ($\cos^2 \theta = 0.884$) is low and is probably not as good as the other coefficients. There are other indications that this is true as will be seen later when the variation of the true coefficients with temperature is discussed.

Of the results for the polycrystalline metal but little can be said. Observations on the dilatation of such material have been reported by a number of investigators, working mostly between 100°C and the temperature of liquid air. The various values obtained are far from concordant, the linear coefficient between 0° and 100°C varying with the author from $26.3 \times 10^{-6}$ to $39.5 \times 10^{-6}$. Such a variation probably arises almost entirely from the preferred orientation in different directions for different specimens.

Of the numerous measurements on polycrystalline material only those of Schulze[24] and of Souder and Hidnert[25] have been carried above 100°C. Although they do not agree with each other both sets of data are considerably higher than those given in Table VII. From these results it is obvious that the linear thermal expansion of any given specimen of polycrystalline zinc is a function of its history and that it is not possible to assign to the metal an expansion coefficient which will be characteristic of all samples.

This uncertainty is reflected somewhat in the calculation of the coefficient of cubical expansion. As ordinarily made this calculation consists in multiplying the coefficient of linear expansion of polycrystalline metal by three. Such an operation is entirely unjustified in this case and the only reliable method of calculation is to use the coefficients parallel and perpendicular to the principal axis of the crystal. Tentative results for the cubical coefficient based on the observed values of $\alpha_{\parallel}$ and values for $\alpha_{\perp}$ obtained by extrapolation are included in Table X where they are compared with the data of Uffelmann[26] who based his calculation on measurement of the dilatation along three mutually perpendicular directions in a cube of zinc. His values are approximately 5 percent lower than the authors which is satisfactory agreement considering the uncertainties involved in both sets of data.

When taking up the question of the allotropy of zinc it is evident from Figs. 7 and 9 that while there do exist certain anomalous features in the expansion of some of the specimens, there is nothing which can be justifiably used as evidence for the existence of more than one form in the temperature interval investigated. This confirms the conclusions of Souder and Hidnert,[25] and of Van de Putte and Thyssen[27] drawn from studies of the expansion of polycrystalline metal. It is, however, at variance with the dilatation experiments reported by Losana[28] and by Uffelmann.[26]

[24] Schulze, Phys. Zeits. **22,** 403 (1931).

[25] Souder and Hidnert, Sci. Papers Bur. Standards **20,** No. 522, 1661 (1925). The reference given in the International Critical Tables, **II,** 478 reference 70 appears to be in error. It should read Sci. Paper 522, not 497.

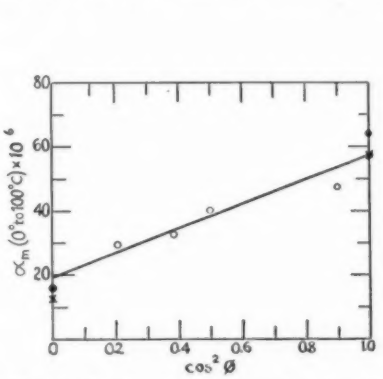
[26] Uffelmann, Phil. Mag. **10,** (7), 633 (1930).

[27] Van de Putte and Thyssen, Rev. Univ. Mines **14,** 49 (1927).

[28] Losana, Gaz. Chim. Italiana **53,** 539 (1923).

In view of the fact that the zinc used by Losana was not of purity comparable with that used by Souder and Hidnert or in the present work and that Uffelmann gives no information on the quality of his material it seems logical to ascribe their results to the presence of impurities and it appears safe to say that the dilatation experiments offer no evidence for the allotropy of zinc.

This conclusion is quite in accord with the x-ray data as summarized by Maier,[29] with the results of thermal analysis on pure (99.999 percent) zinc obtained by Anastasiadis and Guertler,[30] with the physical properties as given by Freeman, Sillers and Brandt,[31] and with the electrical properties of single crystals as reported by Hayem.[32] In fact, the increasing amount of data available for zinc of extraordinary purity leaves little room for doubt that zinc exists in but one allotropic form. It is unfortunate, therefore, that this allotropy is treated as a well established fact in a number of handbooks, such as those of Mellor and Gmelin since it may take a very long time to eradicate the impression which they create.



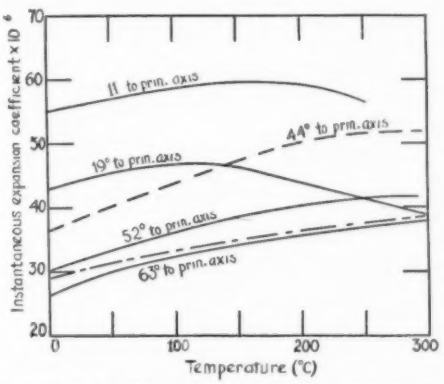




Fig. 8. Cosine-square plot showing the variation of the linear expansion coefficient with orientation in single-crystal zinc. Circles, Austin. Black dots, Grüneisen and Goens, 20° to 100°. Crosses, Bridgman at 20°C.

Fig. 9. The variation with temperature of the true linear expansion coefficient of various orientations in single-crystal zinc. Unbroken curves, Kahlbaum's Zn. Broken curve, spectroscopically pure Zn. Curve with long and short dashes, polycrystalline Zn.

Although there is no evidence for allotropy, there are one or two interesting features about the data which merit comment. In the first place, while the curves of Fig. 7 are good representations of the data, they are nevertheless in the strict sense of the word, smoothed curves. If the actual data are plotted on a scale such that the temperature coordinates are compressed while the length coordinates are expanded, or if one foreshortens the curves by looking down along them, it is observed that the metal has actually expanded in a

[29] Maier, *Zinc Smelting from a Chemical and Thermodynamic Viewpoint*, Bull. Bur. Mines **324** (1930).

[30] Anastasiadis and Guertler, Z. Metallkunde **21,** 338 (1929); also Metals and Alloys **2,** 354 (1931).

[31] Freeman, Sillers and Brandt, Sci. Papers Bur. Stands. **20,** No. 522, 661 (1925).

[32] Hayem, Phys. Rev. **38,** 1357 (1931).

series of waves or steps. These disturbances are not large but they do appear to be real and they indicate that the expansion is not strictly a regular change in dimension but takes place through a series of steps.

These shifts are not sudden as were some observed in magnesium and already commented on but are gentle undulations in the length-temperature curve. They were noticed first in zinc because only one set of measurements was made on each sample but when the data on the other metals were reexamined it was found that the same phenomenon had occurred in the individual runs for them but had been lost in the composition of the data for the several runs since the steps do not follow a regular pattern but seem to be entirely haphazard.

Whether this behavior comes about through a non-uniform release of stresses, as appears to be the case with the relatively sudden shifts in magnesium, or whether it is something characteristic cannot be definitely stated. There are probably some who might suggest that it is in some way connected with the mosaic structure of metals now so fashionable although the relation is certainly not evident. Regardless of speculation as to its origin, however, it appears to be a subject worthy of further study.[33]

Another interesting feature to be found in the results is illustrated in Fig. 9 which shows graphically the variation of the true coefficients with temperature. All of the curves show a flattening at the higher temperatures which appears to be characteristic although it is possible that it may be nothing more than a softening of the metal at the higher temperatures. The latter explanation is certainly feasible because the metal is very soft even at room temperature yet the former one is supported to some extent by the fact that the observations of Souder and Hidnert made with an apparatus using an entirely different principle and probably but little affected by such a softening show a similar tendency. It is unfortunate that data on cooling could not be obtained since such information would demonstrate whether or not the effect is reversible.

The curves for $\theta = 19°$ and for $\theta = 0°$ however, show an actual decrease in the coefficient, the effect being marked in the first case but appearing only slightly in the second. It has already been mentioned that the coefficient for the direction $\theta = 19°$ does not fit in with the other data on the $\alpha$ *vs.* $\cos^2 \theta$ diagram so that indications are strong that this crystal is not of a quality comparable with the others. Since the spectroscopically pure zinc shows a flattening but no decrease it is probable that the latter is to be laid to presence of impurities.

As a matter of fact, admitting the presence of impurities, the effect produced is precisely what one would expect from reasoning similar to that employed by Anastasiadis and Guertler. If the crystal contains even very small amounts of metals such as lead and cadmium it is actually a ternary system and, in spite of the fact that the specimen is a single crystal, some liquid, even though almost infinitesimal in amount, will form at a tempera-

[33] It has been suggested by Mr. A. Q. Tool in a personal communication to the author that this wave effect may be the result of the differential expansion of samples and plates.

ture below the melting point of pure zinc. If the quantity of liquid is extremely small it is possible that no effect detectable with ordinary apparatus will be produced. On the other hand, if an appreciable amount of liquid is formed the single-crystal structure may easily be destroyed and in its place there will be a number of small crystalline units associated with the liquid. This is equivalent to a rather stiff mush which would become softer as the melting point of pure zinc is approached and which would tend to flow at all temperatures at which it exists. This would appear in the measurements as a decrease in the true coefficient.

In conclusion it is interesting to see how closely the data fulfill the prediction of Grüneisen that the heat capacity ($C_p$) should be proportional to the volume expansion coefficient ($\beta$).[34] Grüneisen and Goens[22] have shown that this relation holds to a very fair degree of approximation for zinc at low temperatures, i.e., below 0°C, but it has not, so far as the author is aware been tested at high temperatures.

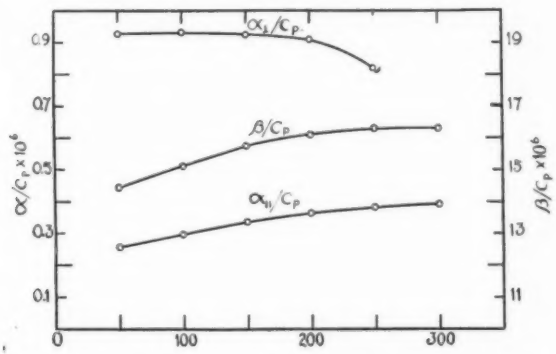



Fig. 10. Diagram showing variation with temperature of the ratio: expansion coefficient/heat capacity for zinc.

Fig. 10 shows the ratios $\beta/C_p$, $\alpha_\parallel/C_p$ and $\alpha_\perp/C_p$ plotted against temperature. In making the calculations on which the figure is based the coefficients as given in Tables IX and X were used and $C_p$ was obtained from the equation $C_p = 5.29 + 2.64 \times 10^{-3}\, T$ cals per g atom as given by Maier.[35]

TABLE X. *True cubical expansion coefficient of zinc.*

| | $\beta \times 10^5$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $t$°C | 50 | 100 | 150 | 200 | 250 | 300 | 350 |
| Calcd. from $\alpha_\parallel$ and $\alpha_\perp$ | 8.9 | 9.50 | 10.10 | 10.56 | 10.91 | 11.13 | — |
| Uffelman ($\alpha_1+\alpha_2+\alpha_3$) | — | — | 9.69 | 10.10 | 10.40 | 10.94 | 11.18 |

It is apparent that the proportionality is only roughly approximate, the deviations from the theoretical straight line horizontal to the $T$ axis being too

[34] Grüneisen, Ann. d. Physik **39**, 257 (1912).
[35] Maier, reference 29, p. 11.

large to be ascribed to experimental error; therefore, the relation of Grüneisen as stated does not hold for the macroscopic expansion of zinc at high temperatures.

This conclusion is in agreement with that to which one is lead by a study of the results of Roberts[36] on the expansion of single crystals of Bi. His data show that the ratio of the macroscopic expansion coefficients to the heat capacity is not constant but decreases slowly with temperature.

He also found that between a temperature of approximately 230°C and the melting point (270°C) the expansion coefficient diminished rapidly. This effect, which was entirely reversible, he ascribed to the presence of impurities.

Unfortunately in the case of zinc the difficulties encountered with the vaporization of the metal prevented the extension of the measurements much above 300°C so that the question of the possible existence of a relatively sudden drop in the expansion coefficient just below the melting point cannot be settled at this time. There are indications, which have already been mentioned that the expansion coefficient may decrease above 300°C but no data on cooling are available with which to check the reproducibility of this tendency.

It should be mentioned in this connection that the lattice expansion of single-crystal Bi has recently been determined by Goetz and Hergenrother[37] using x-rays. They found that in contrast to the macroscopic expansion the lattice expansion was regular from room temperature to the melting point and moreover that if one uses the lattice expansion coefficient the Grüneisen relation is also valid up to the melting point. It would be interesting to have similar data on zinc to see whether the lattice expansion would again give closer agreement with theory than does the expansion of a macroscopic crystal.

## Conclusions

A vacuum apparatus for the measurement of linear thermal expansion has been tested under a variety of conditions and appears to give excellent results for metals with relatively low vapor pressures. It has been found possible to maintain a vacuum of several microns at the maximum to temperatures above 900°C and under these conditions oxidation does not occur to any appreciable extent.

With metals having a high vapor pressure the deposit of condensed vapor on the cool parts of the apparatus is often a serious handicap. The vaporization of magnesium and zinc in vacuum was much greater than was to be expected from their behavior in air at one atmosphere. This difference leads one to wonder whether the true vapor pressure of metals at temperatures of but a few hundred degrees is not greater than is commonly supposed.

A study of the expansion of single-crystal zinc shows no evidence for the existence of more than one polymorphous form of zinc.

[36] Roberts, Proc. Roy. Soc. **A106,** 385 (1924).
[37] Goetz and Hergenrother, Phys. Rev. **40,** 643 (1932).

It is a pleasure to acknowledge the author's indebtedness to Professor Bridgman for his kindness in supplying the single crystals of zinc and to the New Jersey Zinc Company for supplying the pure zinc and for the analytical work described. He wishes also to express his appreciation to G. E. Merritt for the valuable advice which he gave on the design and construction of the apparatus and to acknowledge the help of R. H. H. Pierce, Jr., who assisted in making the measurements.

# THE EDITOR'S COLUMN

## The Wave Nature of Atoms

IT IS a most peculiar coincidence in the recent history of physics that at about the same time that Professor L. de Broglie in Paris was elaborating his ideas on the wave nature of the electron, Davisson and Germer in New York were working on their famous experiment which soon was to become one of the most important and substantial props to de Broglie's ideas; that is, their experimental demonstration of the diffraction of electrons by crystals. Since that time electrons have been diffracted in every conceivable manner and overwhelming evidence produced that a great many of their properties can be explained on the very simple assumption that they behave like waves having a wave-length

$$\lambda = h/p$$

where $h$ ($6.54 \times 10^{-27}$ erg sec.) is Planck's constant and $p = mv$ is the momentum of the electrons. Not long after it was shown that protons and atoms also behave in just the same way if they are employed in the diffraction experiments.[1] This may seem somewhat more surprising since an atom is a rather complex dynamical system, but after all it is true that the experiments are not overly precise. Just as one can predict the simple *positions* of diffraction fringes for ordinary light from a knowledge of the wave-length alone but must have a detailed theory for the prediction of the *intensity distribution* in the fringes, so will it undoubtedly be found that as the experiments on the diffraction of particles become more precise, more and more complex theories will be required for the explanation of all of their detailed results.

The experimental work on the diffraction of atoms has already progressed by leaps and bounds and now constitutes a well-defined field of research.[2] In a recent paper in the *Physical Review* a report by R. M. Zabel describes some of the latest work done at the University of Iowa, the principal object being to investigate the influence of the crystal surface on the observed diffraction patterns. Helium, neon and argon atoms were used for diffraction from sodium chloride crystals grown both naturally and in the laboratory. In preparing the fresh crystal surfaces for the experiments various techniques were used, some being cleaved in dry or moist air, and some in an atmosphere of hydrogen. It is well known from other work on crystals that the properties of a surface are influenced tremendously by the gases which are adsorbed on it, and one would naturally expect this influence to be very apparent in these diffraction experiments, an expectation which has been fully verified. The presence of moisture during the preparation of the crystal was found to produce a marked effect both in reducing the intensity of the diffraction patterns and also in changing their form, the patterns from such surfaces showing a displacement of the diffraction "fringes" which was interpretable as due to an increased lattice spacing in the crystal. The degree of roughness of the surface as measured by its reflecting power for light was also found to be an important factor in determining the intensity of the patterns, that is, the number of atoms which are diffracted. Detailed examination of these effects may show that atomic diffraction can be used for the investigation of surface conditions just as electron diffraction has already been so used.

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[1] A. J. Dempster, Phys. Rev. **34,** 1493 (1929).

[2] Esterman *et al*, Zeits. f. Physik **73,** 348 (1932) and many previous papers from the molecular ray laboratory at Hamburg. T. H. Johnson, Phys. Rev. **37,** 847 (1931). A. Ellett and R. M. Zabel, Phys. Rev. **37,** 1112 (1931). R. M. Zabel, Phys. Rev. **42,** 218 (1932).

## The Mass of the Hydrogen Isotope $H^2$

SINCE the announcement last year by Urey, Brickwedde and Murphy of the experimental discovery[1] of an isotope of hydrogen having a mass of approximately *2* atomic units, an accurate determination of its mass

[1] Urey, Brickwedde and Murphy, Phys. Rev. **39,** 164 (1932).

has been in order. Reports of two such determinations are given in recent papers in the *Physical Review* by Bainbridge[2] and by Hardy, Barker and Dennison.[3] The experimental arrangements are of quite different sorts, that of Bainbridge being the mass-spectrograph while the latter authors have examined the band spectrum of the molecules $H^2Cl^{35}$ and $H^2Cl^{37}$. Unfortunately, while the masses found by the two methods are nearly the same, the discrepancy seems to be too large to be within the estimated probable error of the experiments. Part of the difficulty arises from the uncertainties in our present knowledge of the masses of atoms other than $H^2$ since the mass-spectrograph determinations involve a knowledge of the masses of $H^1$ and He while the band spectrum determinations involve the masses of $H^1$, $Cl^{35}$ and $Cl^{37}$. According to Bainbridge, if the mass of $H^1$ is taken to be 1.00796 then that of $H^2$ is

$2.01351 \pm 0.00018$ (mass-spectrograph)
$2.01403 \pm 0.00010$ (band spectrum),

the discrepancy being outside the probable error given for either determination. It is not possible as yet to conclude from these results whether the $H^2$ nucleus is made up in the form

$$H^2 = 2H^1 + \text{electron}$$

or in the form

$$H^2 = H^1 + \text{neutron},$$

as any measured mass defect could be attributed to first a loss in energy in the formation of the neutron and a further loss on formation of $H^2$ or to a loss in the formation of $H^2$ from two protons and an electron in a single process.

The band spectrum measurements yield an estimated relative abundance of 35,000 atoms of $H^1$ to 1 of $H^2$ in ordinary HCl gas, a concentration of $H^2$ much less than was indicated by the first experiments of Urey and his collaborators, who estimated about 4000 to 1.

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[2] K. Bainbridge, Phys. Rev. **42**, 1 (1932).

[3] J. Hardy, E. Barker and D. M. Dennison, Phys. Rev. **42**, 279 (1932).

## Relativistic Quantum Mechanics

IN PREVIOUS notes in these columns[1] we have reported various aspects of the work which is being done to effect a fusion of the three major theories of present day physics, that is, quantum mechanics, electrodynamics and relativity. It is of interest to note in a general way that this problem is being attacked from two quite different points of view; *viz.*, there is the school of relativists which is attempting a unified theory of relativity and electrodynamics along lines closely akin to pre-quantum dynamics, and there is the newer school of quantum mechanists which is attempting more particularly the fusion of quantum mechanics with electrodynamics. There is some overlapping between them as in the Heisenberg-Pauli field theory which unfortunately has not been as successful as was hoped at first, and in the Dirac theory of the electron which connected wave mechanics and relativity but did not include a true electrodynamical theory. It is desirable to keep in mind the fact that there are really three (at present) distinct theories which require assimilation into the unified theory which remains to be invented.

While the difficulties involved in fusing these three theories are many and complex it is perhaps fair to say that the two most serious troubles from a physical point of view are, first, the introduction into quantum mechanics of a general method for the calculation of the mutual interactions between charged particles, and second, the finding of an interpretation in quantum mechanics for the relativistic concept of the *proper time* which is associated with each particle. The nature of the first difficulty will be explained in more detail in the following paragraphs with some discussion of the ideas which have been developed for getting around it, but for the second difficulty there seems to be no answer at present.

On inspecting these two snags in the path of theoretical physics one finds that both are intimately related to the one feature of quantum mechanics which separates it most radically from any form of classical dynamics. This feature is the absence in modern quantum theory of any true and unique conception of *velocity*. It is embodied in the principles of quantum mechanics that it is impossible to know and follow the details in the motion of

[1] Editor's Column, Physics **1**, 208 (1932); **1**, 340 (1932); **2**, 101 (1932).

an individual particle. That this fact leads at once to a fundamental difficulty with respect to the concept of *proper time* is clear. For according to the ideas of the special relativity theory there is a proper time appropriate to each individual particle; i.e., "time" as measured in a coordinate system moving with that particle. The relation between the proper times of two particles is such as would be given by the Lorentz transformation connecting them, a relation which requires for its statement the *relative velocity* of the two particles. Hence the elimination of the idea of velocity in quantum mechanics leaves one at a loss as to the relationship between the proper times of the various particles.

But how does this also interfere with the calculation of the mutual interactions between two particles? Let us reflect for a moment on the procedure used for calculating such interactions in classical electrodynamics. Consider two electrons $\alpha$ and $\beta$ each moving along a definite path in a definite and predetermined way. The forces acting on $\alpha$ at a given instant $t$ due to $\beta$ are determined by the values of the electric and magnetic fields at the position occupied by $\alpha$ at the given instant, due to the presence of $\beta$. But since electromagnetic fields are propogated with a finite velocity the radiation from $\beta$ which acts on $\alpha$ at the instant $t$ left $\beta$ at some earlier time, and when exact calculations are made it is found that one must know all of the details of the motion of $\beta$ in order to calculate its action on $\alpha$. Similarly in order to find the reaction of $\alpha$ on $\beta$ one must know the details of the motion of $\alpha$. It is worth remembering too, that because of the finite velocity of propogation of electromagnetic effects the action and reaction between $\alpha$ and $\beta$ at any given instant are not equal. Since quantum mechanics gives one no information on the details of the motions of electrons in this sense it is clear that such calculations of mutual interactions cannot be made in the new theory.

In all fairness to quantum mechanists it must be noted, though, that the superiority of the classical theory on this head is really not so great as at first appears. For the problem we stated was this: two electrons move along pre-assigned paths in a predetermined manner, to find the interactions. Actually the dynamical problem should have been stated thus: two electrons are set in motion initially in any specified manner, to find the resulting motions. This latter problem is infinitely more difficult to solve than that first stated for now the interactions between the electrons can (and will) influence their motions. The method of solution generally adopted in calculations on such problems in the classical theory was to consider that the influence of the mutual interactions on the motions of the particles was small and could be treated in the nature of a perturbation. The more general dynamical problem was never solved completely.

The position in which the quantum theorist finds himself now is that he really must solve the problem completely before he can give a satisfactory answer to any part of it. Even if one desires to treat the interactions of the particles as a small perturbation one must face the difficulty that in principle he cannot calculate them in the manner which we have outlined for the classical dynamics.

A possible alternative procedure leading one out of the vicious circle can be obtained if one remembers that in the classical electrodynamics of *continuous* charge distributions the electric and magnetic field vectors can be obtained when the charge and current densities are known for all points $x, y, z$ and for all instants $t$. Now quantum mechanics does have general expressions for these quantities which can be calculated when the $\psi$-function is known. It was suggested by Schrödinger in his original work on wave mechanics that these expressions could be inserted in the classical expressions for the field vectors, so that when the $\psi$-function was known the field vectors would also be known. This interpretation of the relation between quantum mechanics and electrodynamics fell into disrepute when the statistical interpretation of the theory was developed, but by returning to it one has at least a possible way of making some progress. The procedure to be followed is: a $\psi$-function is assumed of a form suitable for the initial description of the system *without* interaction. The charge and current densities and the field vectors are then computed by using this $\psi$-function and the interaction obtained just as though one were treating continuous charge distributions using the Maxwell-Lorentz field equations. Then this interaction is employed in connection with the perturbation theory of quantum mechanics to calculte the change in $\psi$, representing the disturbance of the system by the interaction, in close analogy with the corresponding perturbation calculationswhich we outlined above for the classical theory.

The method is rather limited in application since it can be used only for calculating small perturbations to some initially assumed $\psi$-function; there are also other troubles which make it difficult to fit it into a general theory. Nevertheless, in recent papers Møller[2] has given some very interesting applications of the method in calculations on the scattering of $\beta$-particles by atoms, and has shown how it can be fitted in with relativistic ideas and also how it introduces a form of electron spin interaction in a very natural manner. But perhaps the most interesting feature of Møller's calculation is the verification of the fact that the form of interaction which one obtains is not a function of the relative coordinates and spin variables of the electrons alone, but depends in an intimate way on the initial and final quantum states of the system, so that even such a simplified theory does not really free one from the ultimate necessity of determining interaction and motion simultaneously.

In a recent paper Dirac has proposed a scheme[3] for avoiding the primary necessity of stating the character of the interactions between the charged particles *before* commencing the calculations of the quantized states for the system. Briefly described the procedure is to write down the complete wave equation involving the coordinates and momentum operators of the electrons (and protons), their spin coordinates, and appropriate coordinates for describing the electromagnetic radiation field. The $\psi$-function is considered as a function of all of these unknowns. One does not introduce into this equation any interaction of the electrons with each other directly but only terms representing the interactions of the charged particles individually with the radiation field, which Dirac supposes satisfies the Maxwell-Lorentz field equations for free space. The whole system consists of charged particles plus radiation field, in striking analogy with the old idea of "ether-coupled" electrons. To solve this equation, which of course is much too difficult to do actually, is to solve at once the complete dynamical problem for the system particles plus radiation.

On the basis of even such rough arguments as we have been able to present in this note it is clear that we cannot legitimately expect (or want) the complete theory to yield any simple interaction energy between two electrons alone, just as this was found not to be the case in Møller's calculation. Nevertheless Dirac has shown that if the two electrons are required to move along a straight line they behave approximately as though they attract each other with a force proportional to the product of their charges and to the square of the distance between them. It appears that the fact that one gets an attraction is a peculiarity of the one-dimensional problem for in later papers Fock and Podolsky[4] have examined the three-dimensional case (electrons in ordinary space) and have found the usual expression for the Coulomb interaction of the electrons, and in addition have found other terms which are interpretable as due to the interactions of the spins of the electrons. So we may feel sure that although the calculations involved in this latest theory of Dirac are quite complex and involve some mathematical procedures which are not easily justified it is beyond a doubt a very important contribution to the theory.

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[2] E.g., Chr. Møller, Ann. d. Physik (5), **14,** 531 (1932).

[3] P. A. M. Dirac, Proc. Roy. Soc. **A136,** 453 (1932).

[4] V. Fock and B. Podolsky, Sow. Phys. **1,** 798 and 801 (1932).